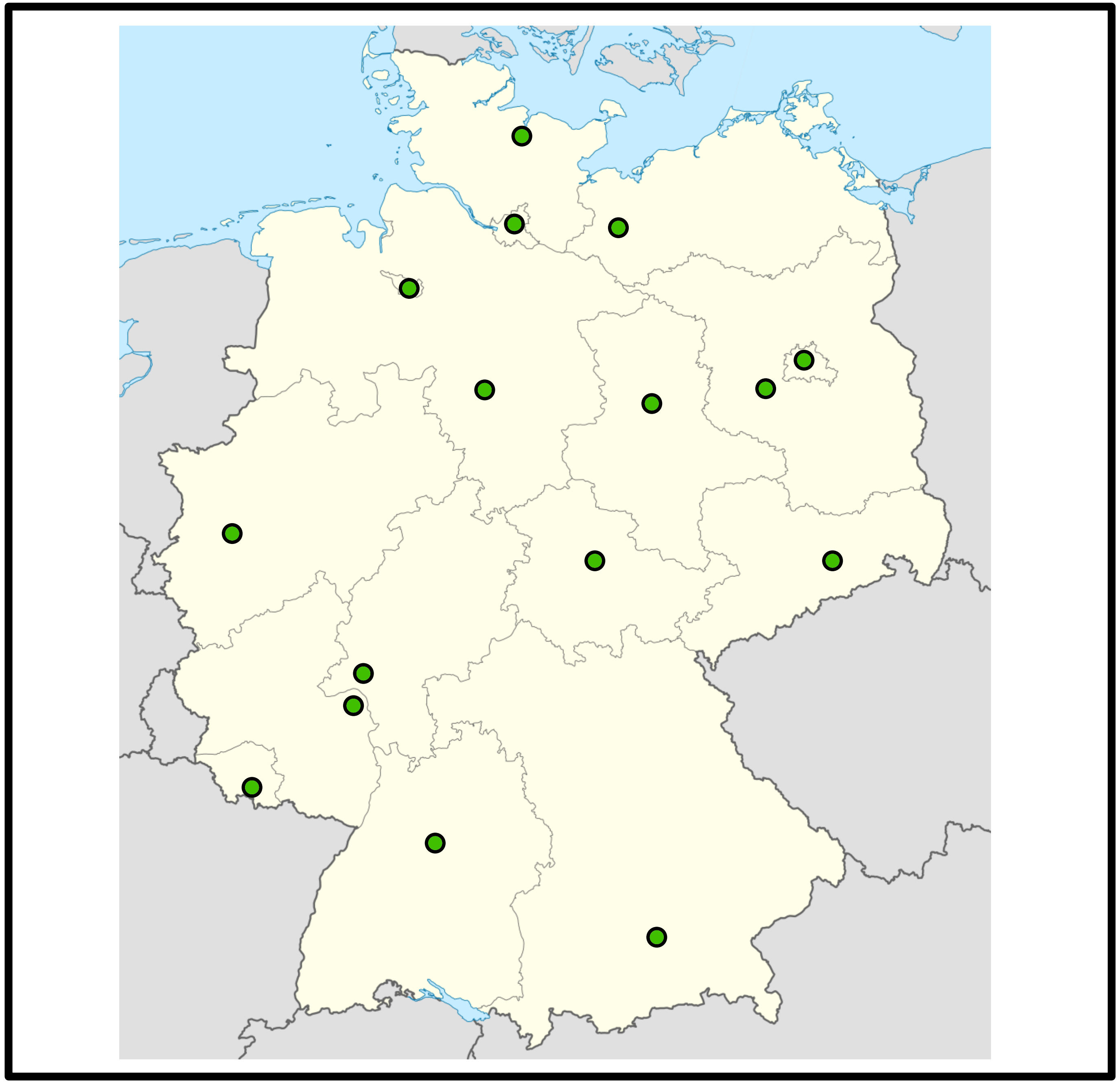
Quadtrees

Geometric Approximation Algorithms

Quadtrees: a simple point-location data structure



Main idea: Use a **tree structure** to be able to quickly find location of point

- nodes represent squares
- recursively subdivide squares into 4 until 1 point per square

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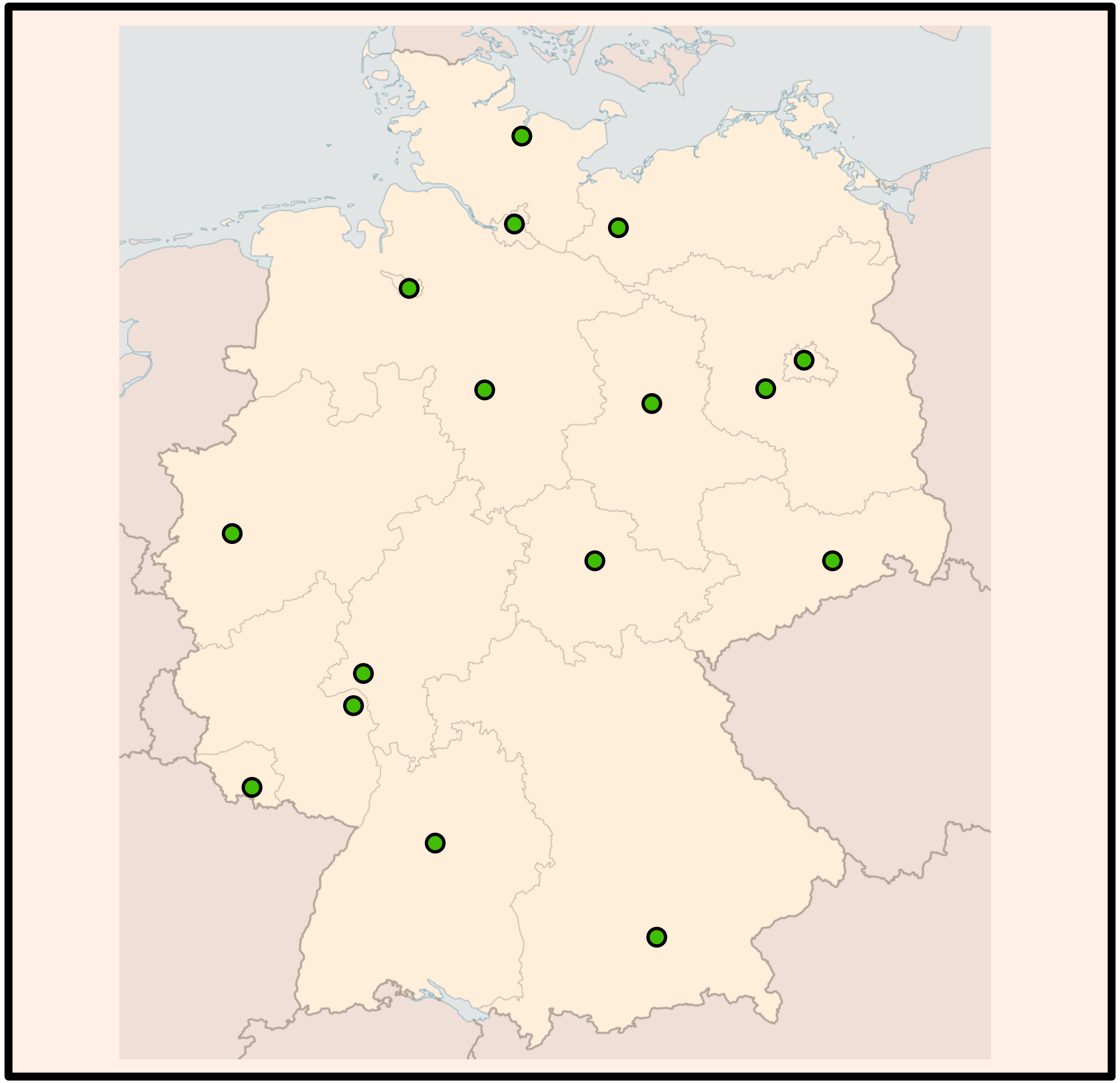

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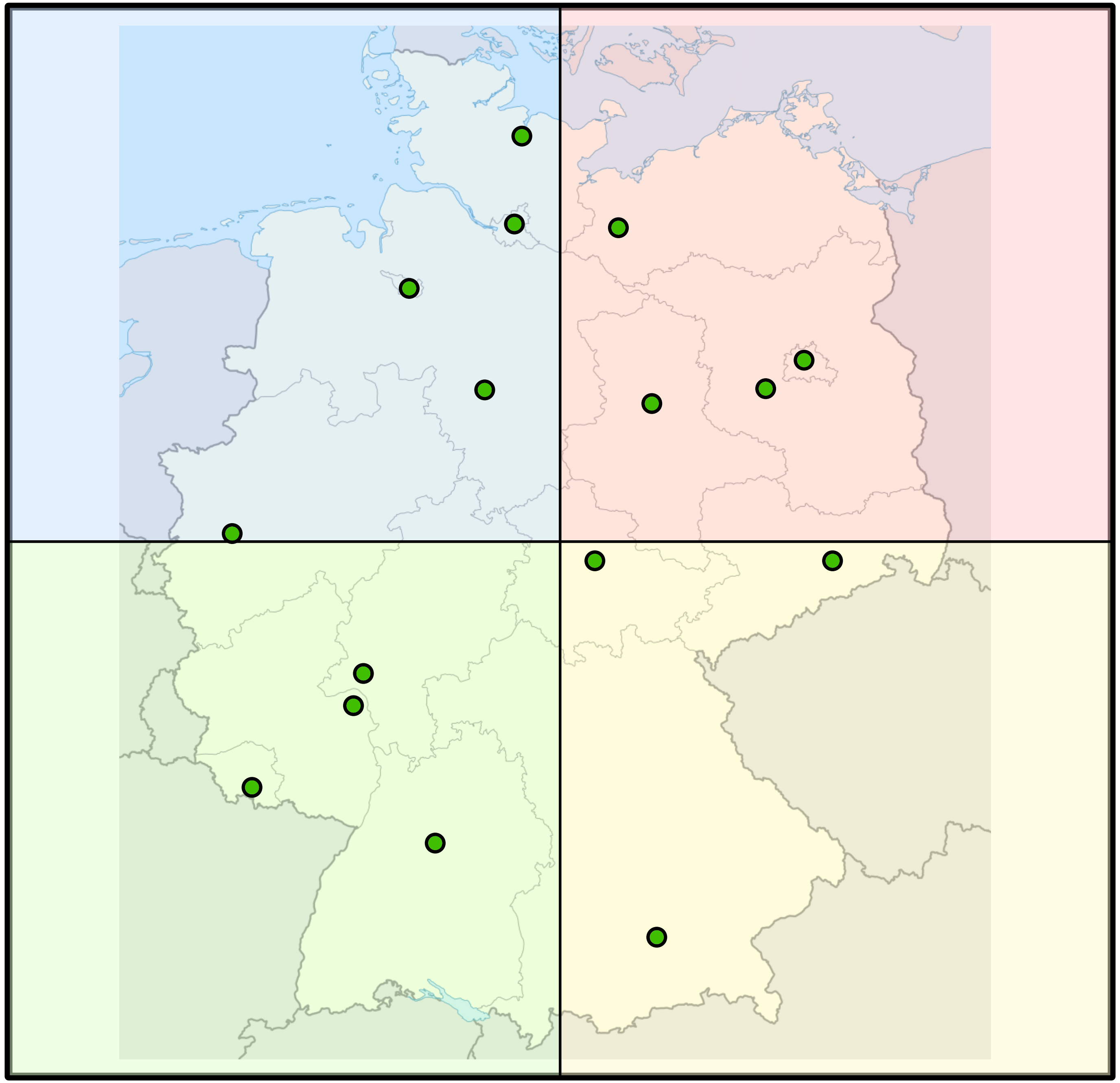

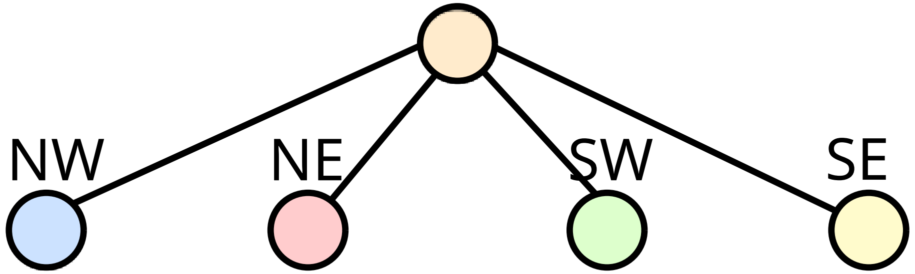

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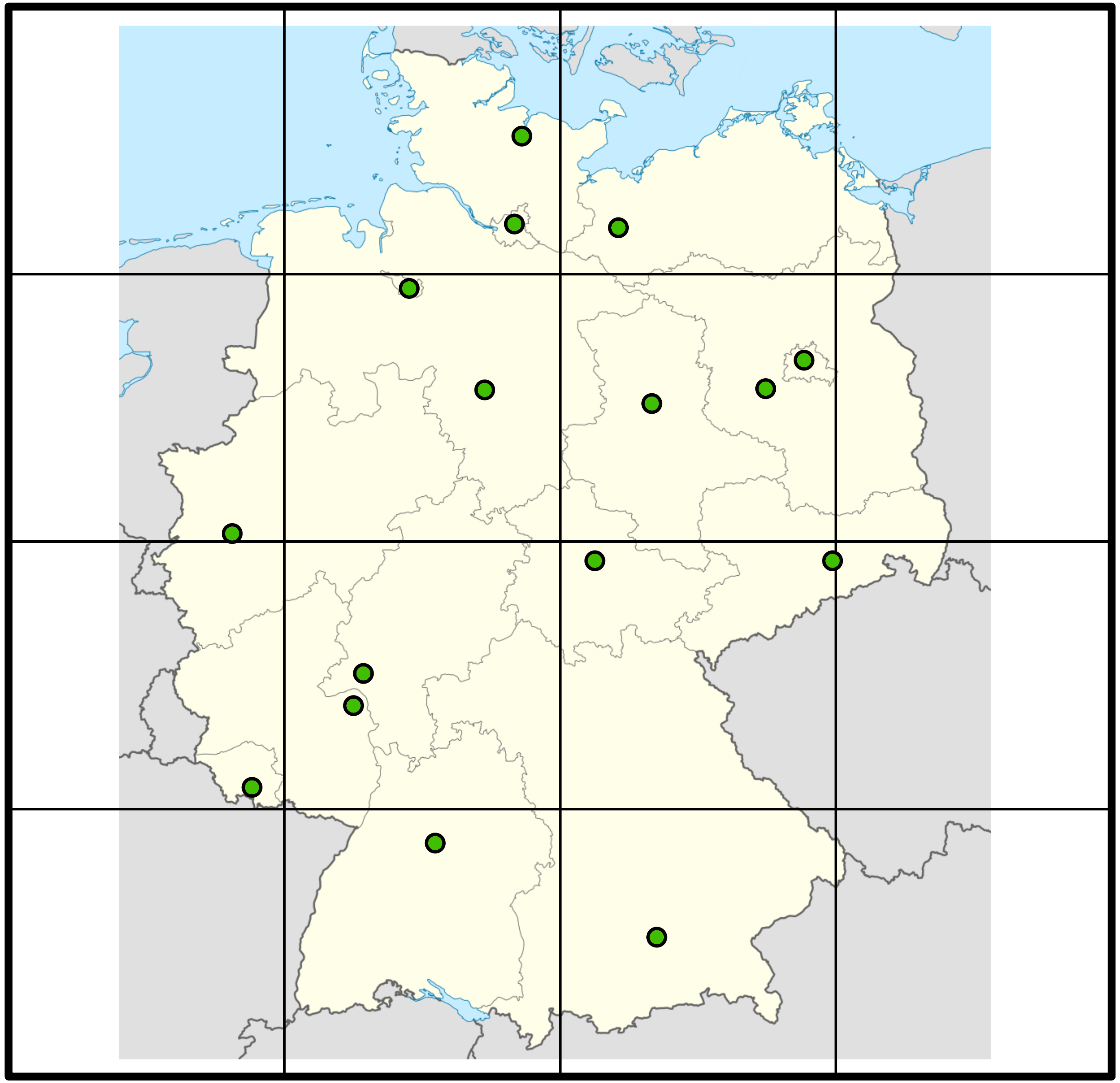

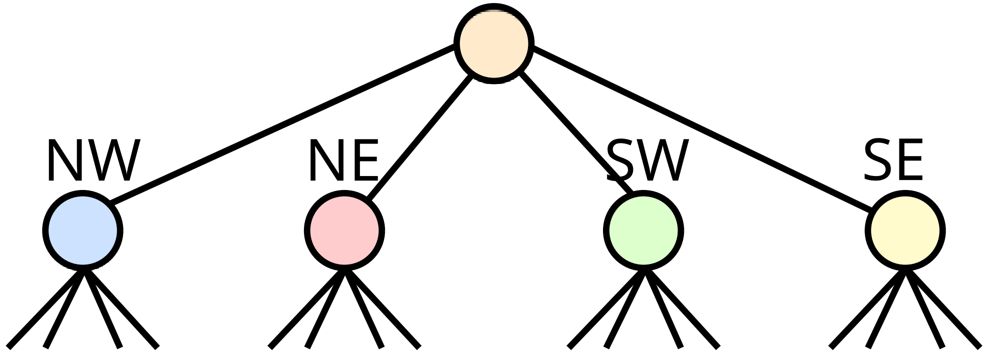

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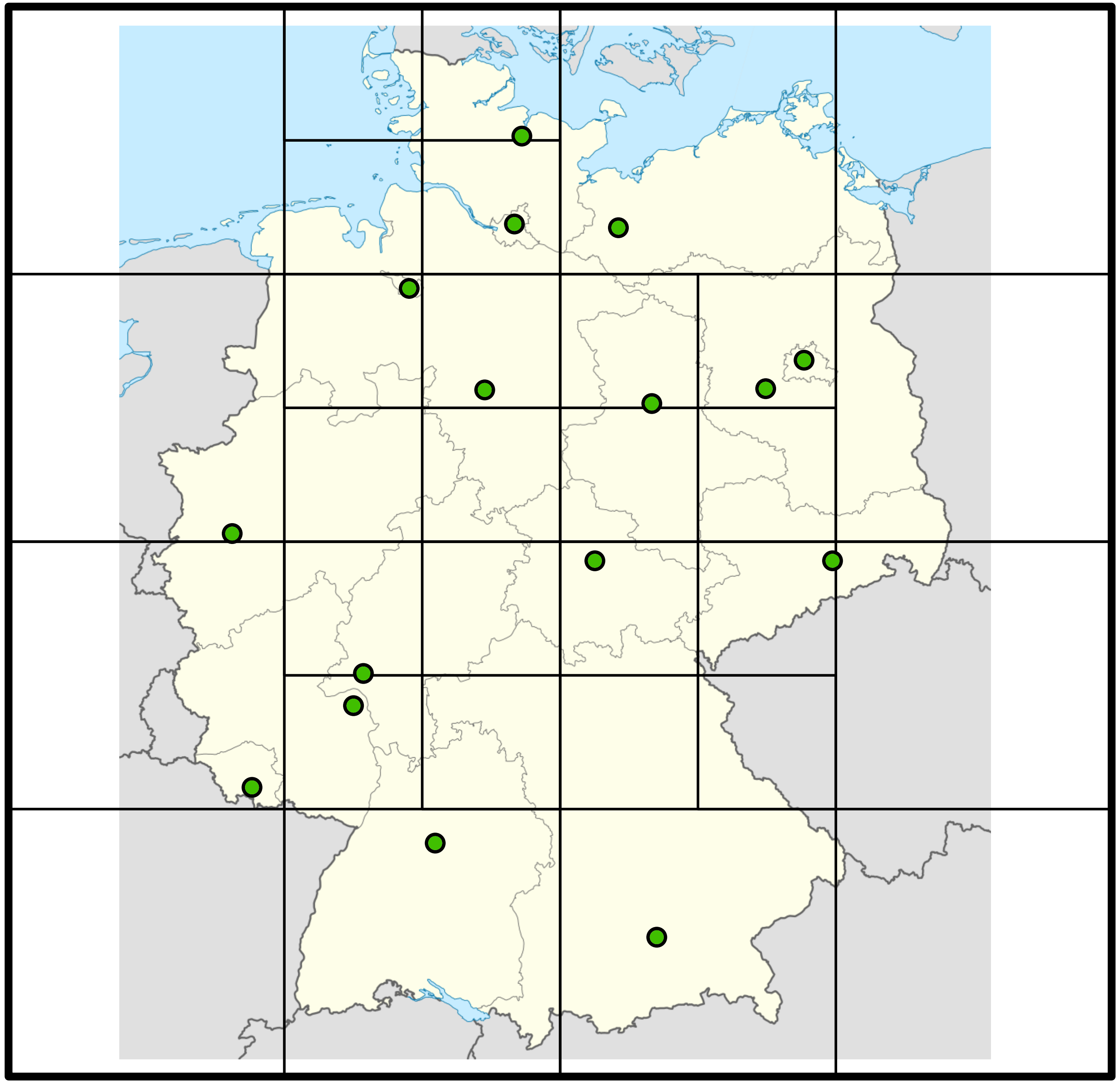

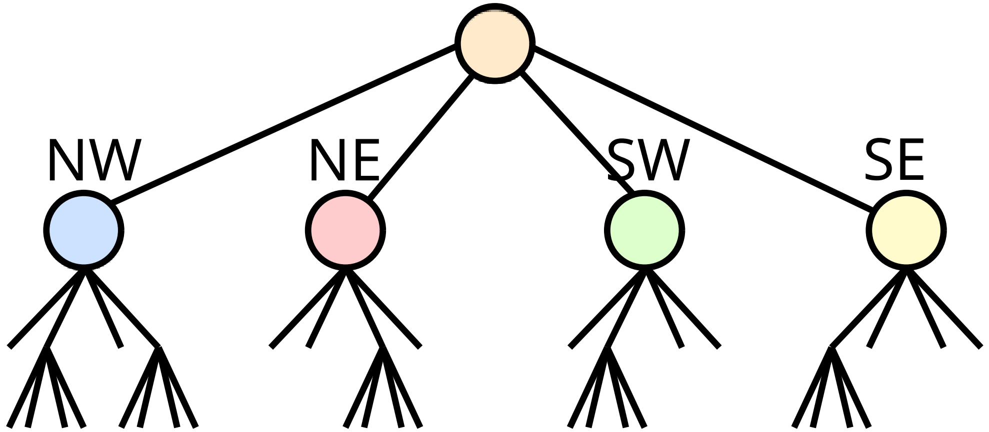

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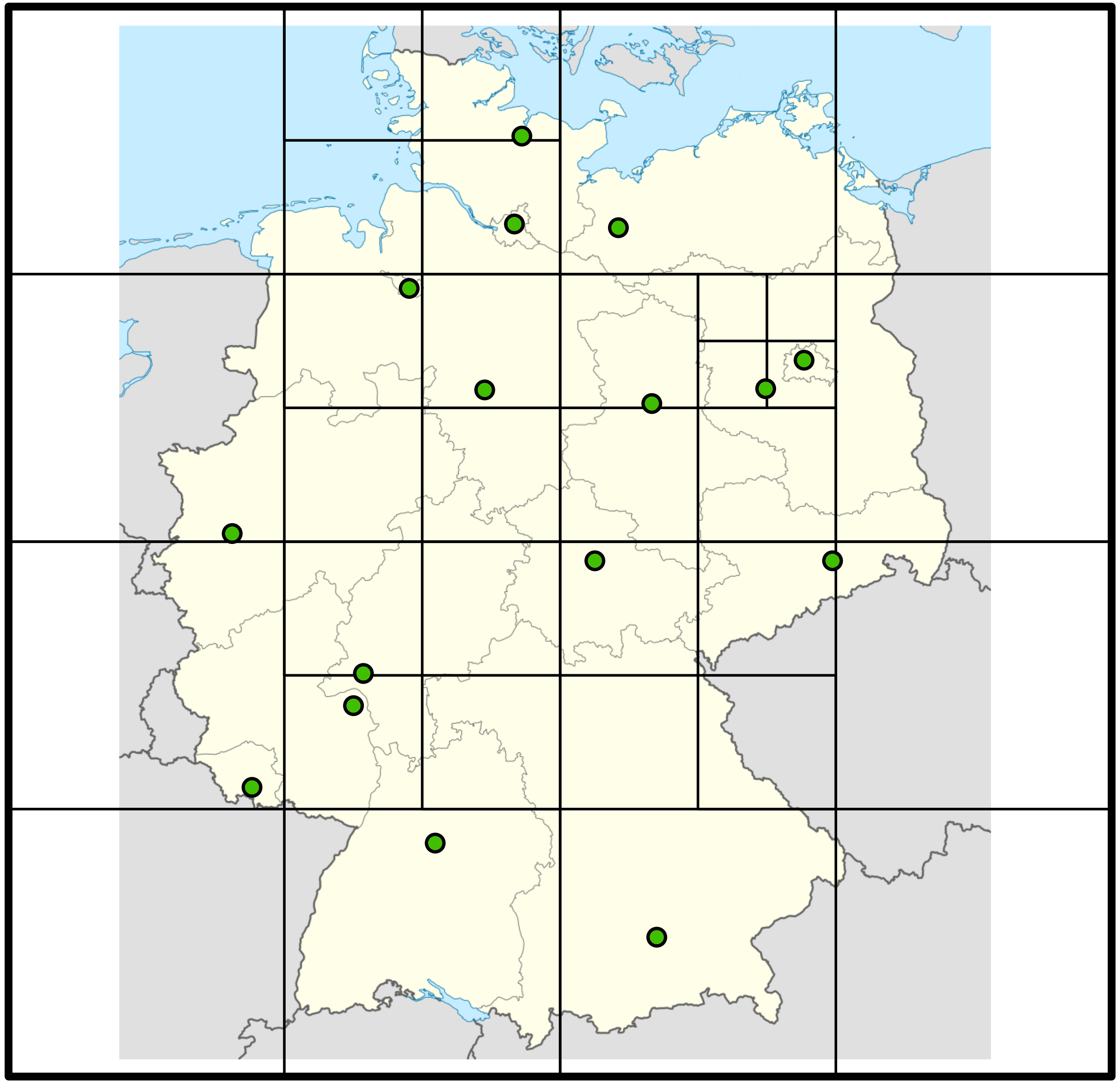

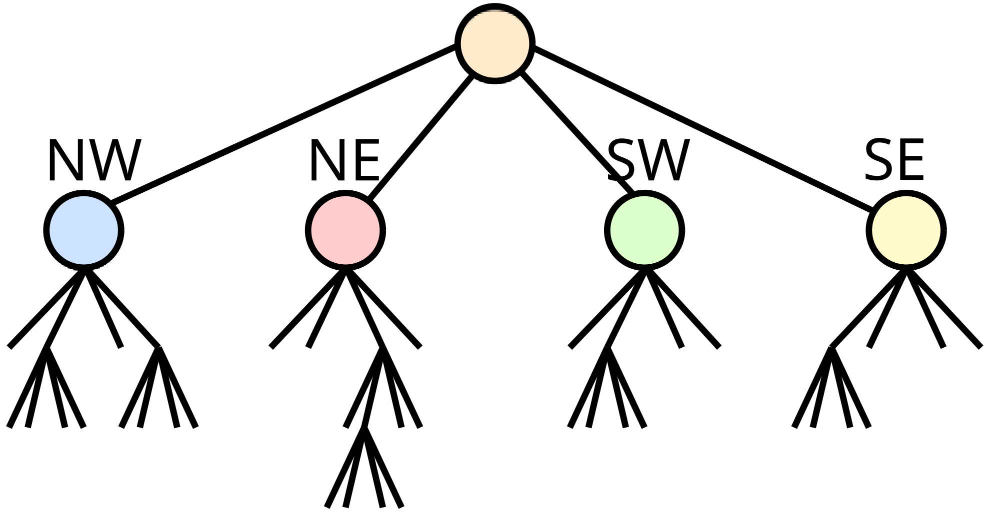

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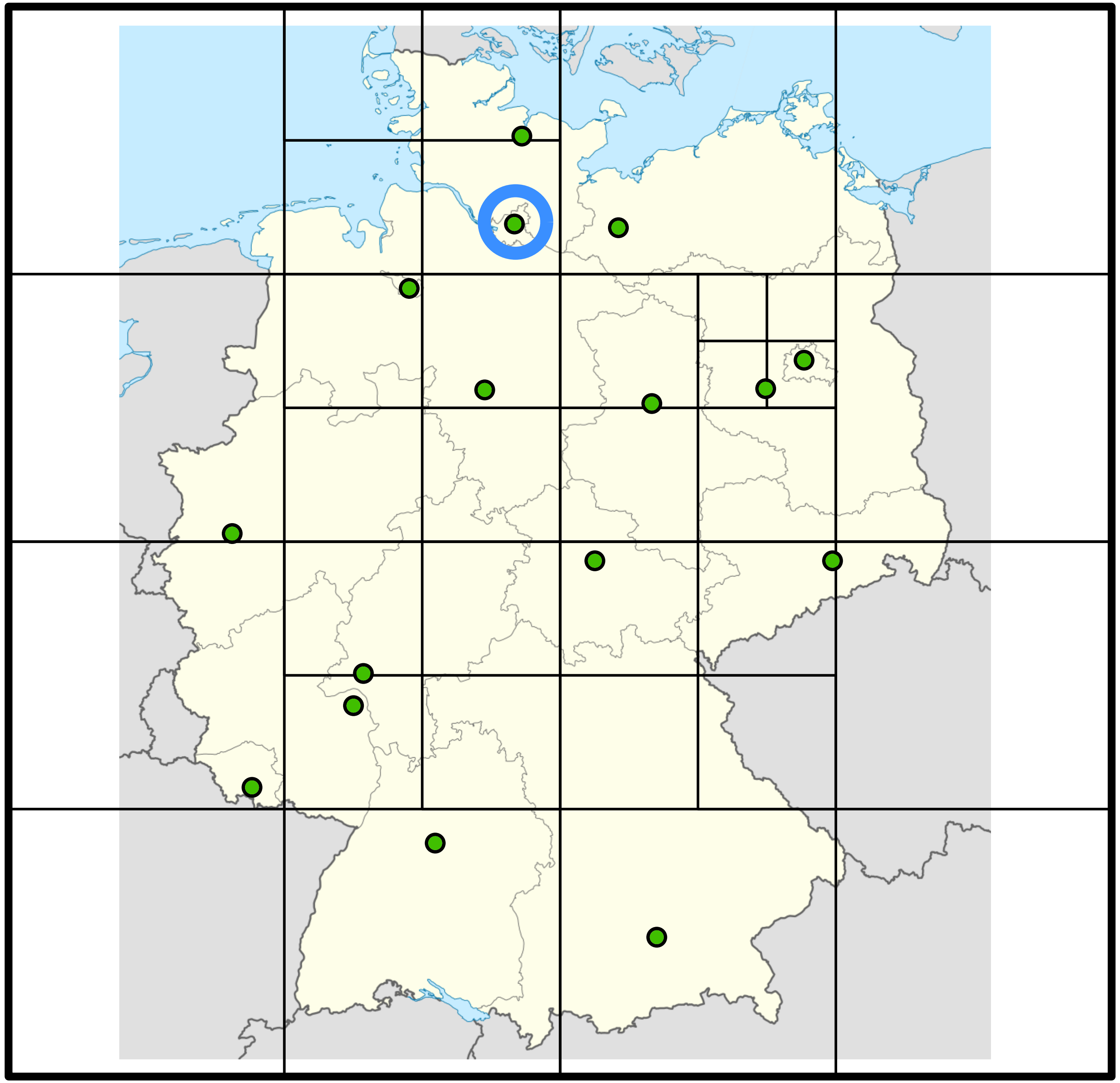

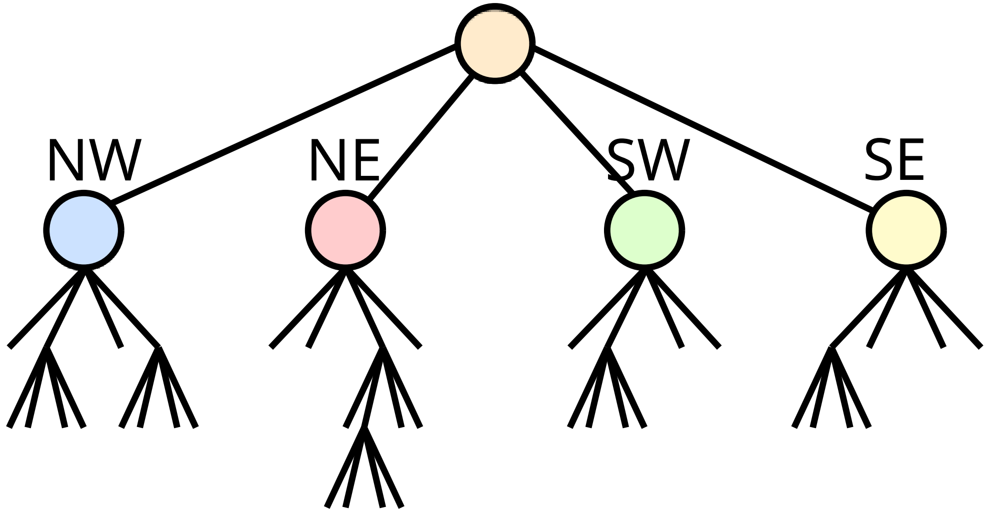

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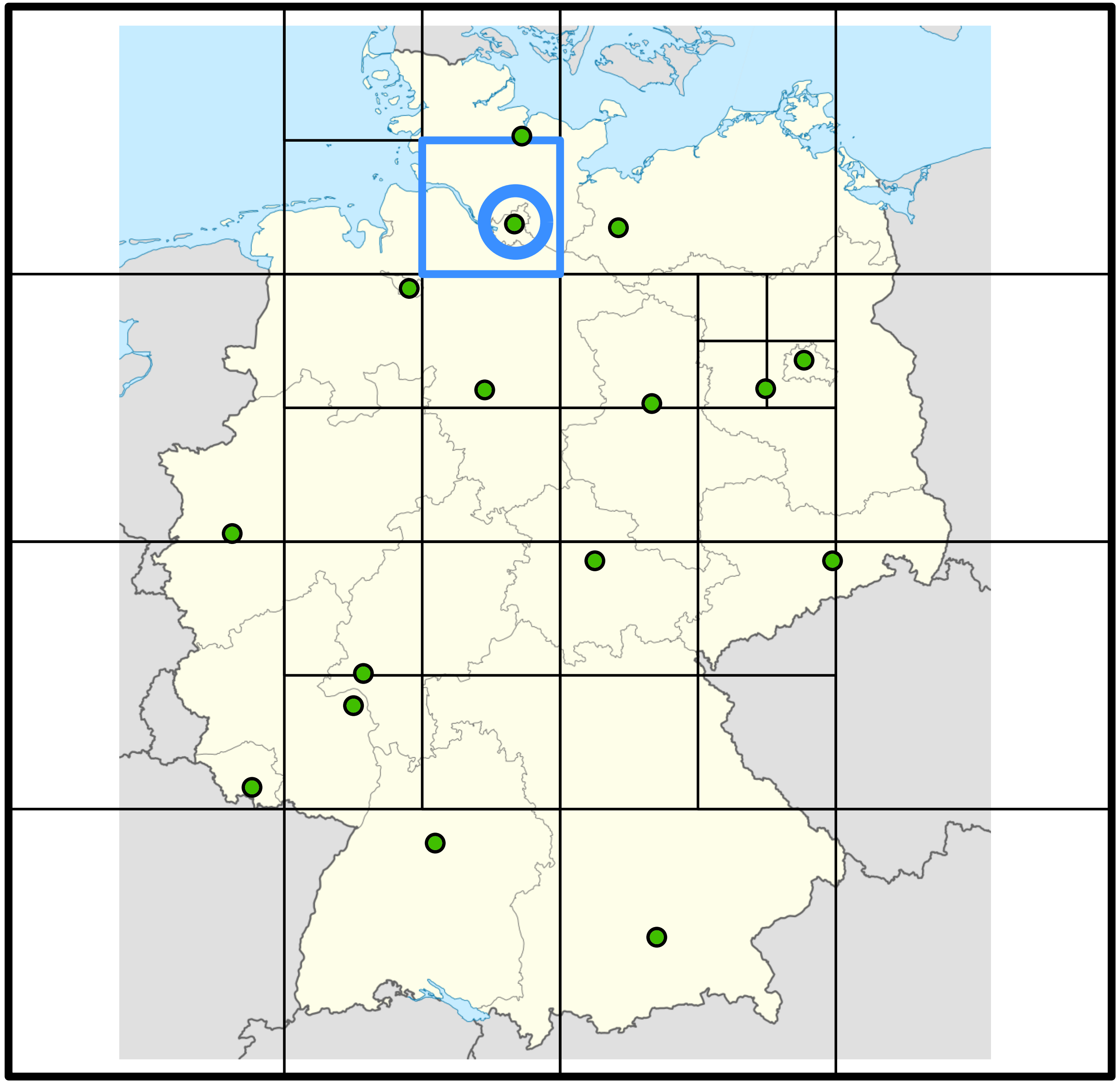

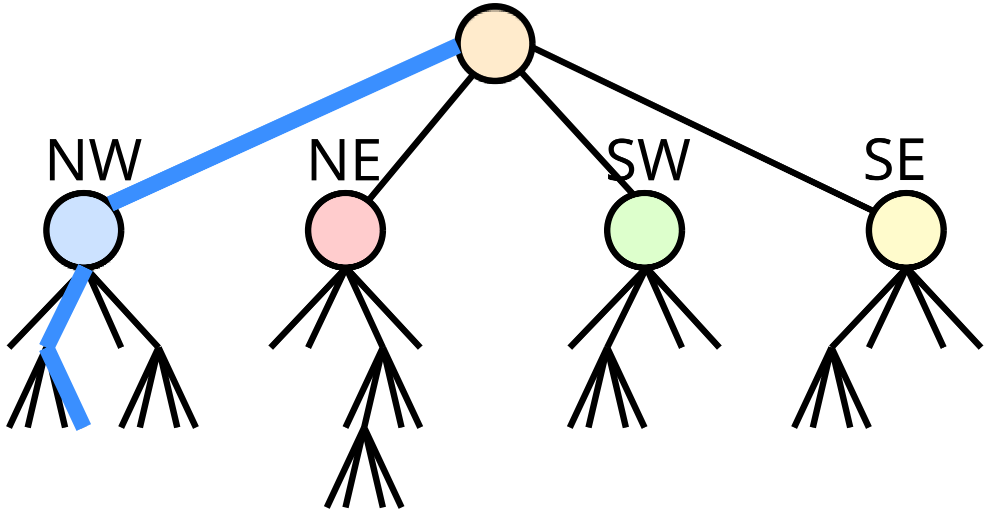

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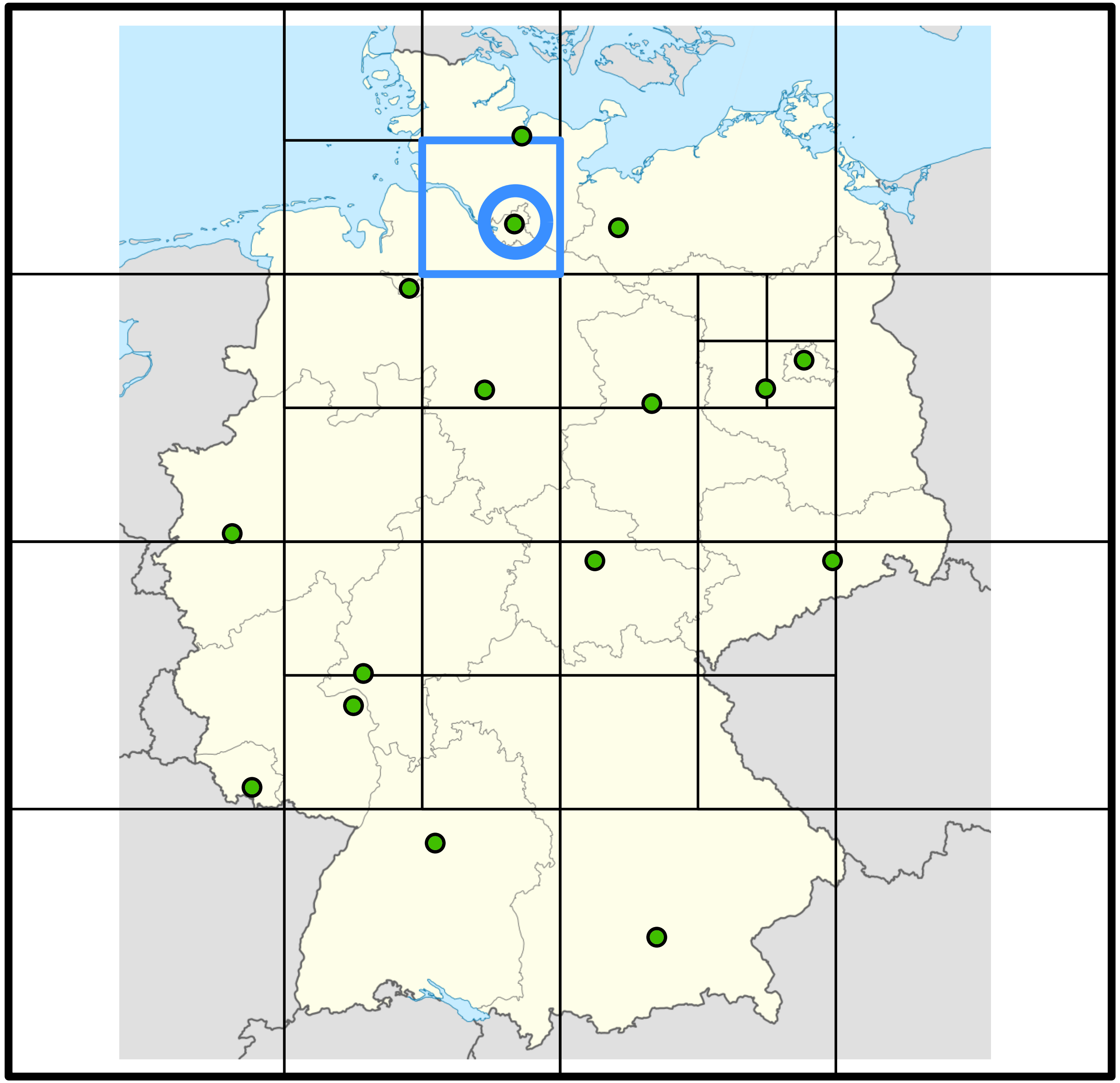

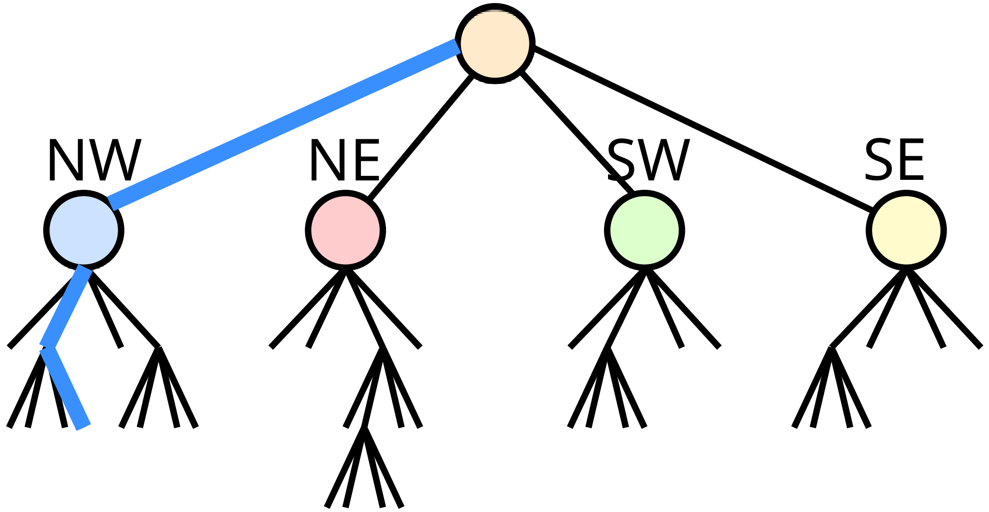

Simple point location: $O(d)$
for n points and quadtree of depth d

(+ check regions overlapping with leaf square)

Quadtrees: multi-scale grids

node v at depth i:

- square $S_v \rightarrow$ side length $= 2^{-i}$
- in grid $G_{2^{-i}}$
- level $\ell(v) = -i$
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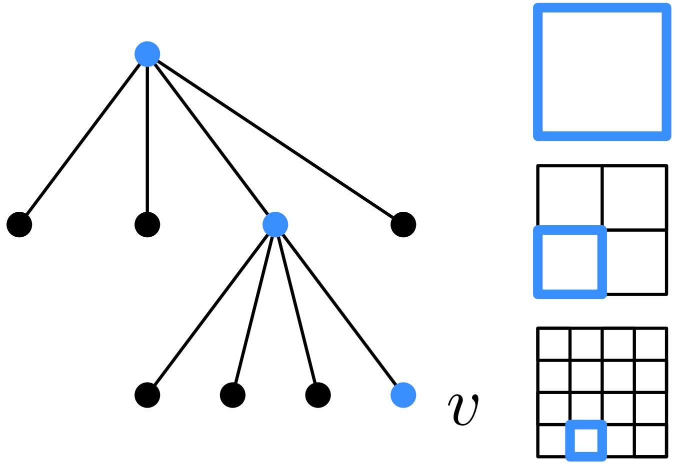

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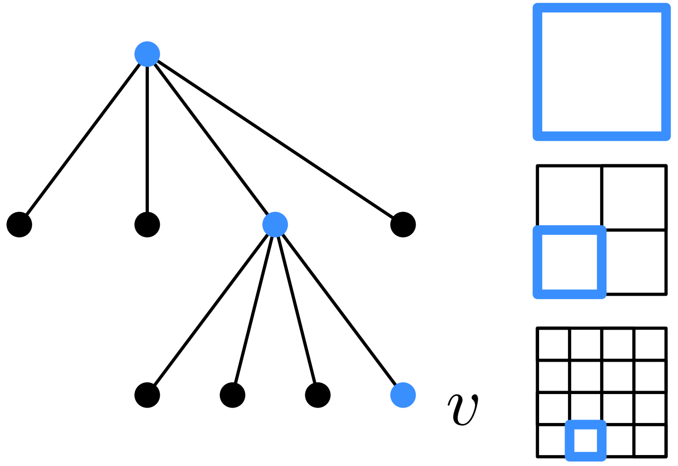
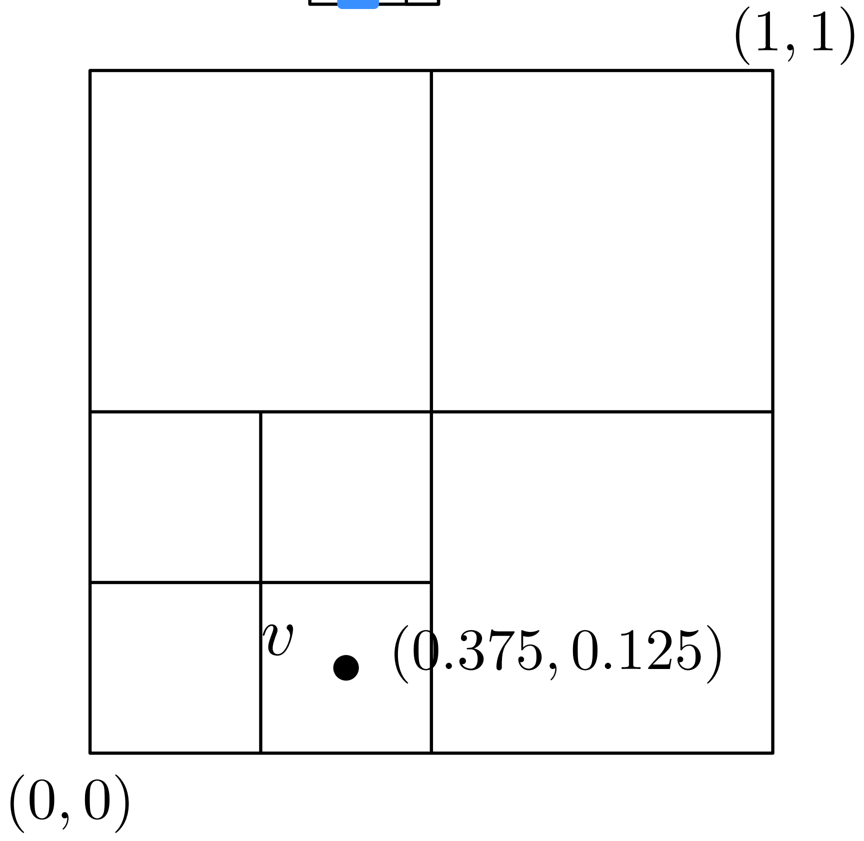


Quiz What is $id(v)$ in this example?



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C (-2,2,2)

D (-2,1,0)

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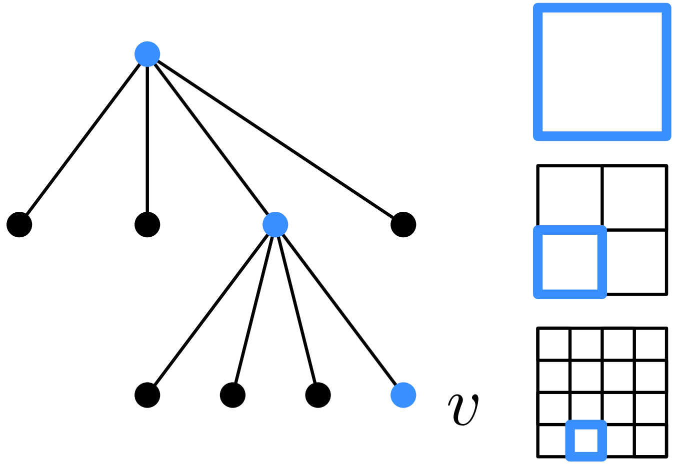

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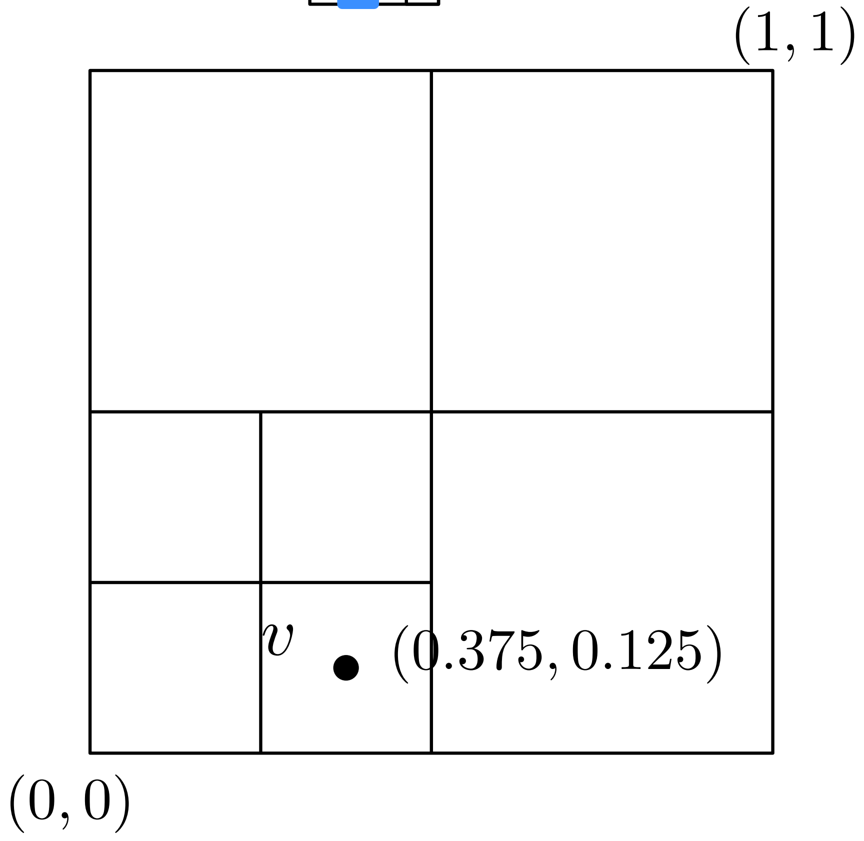

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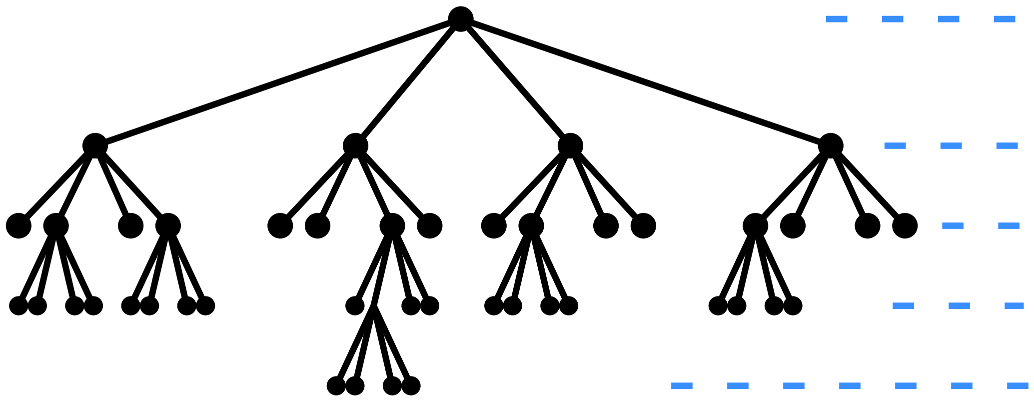

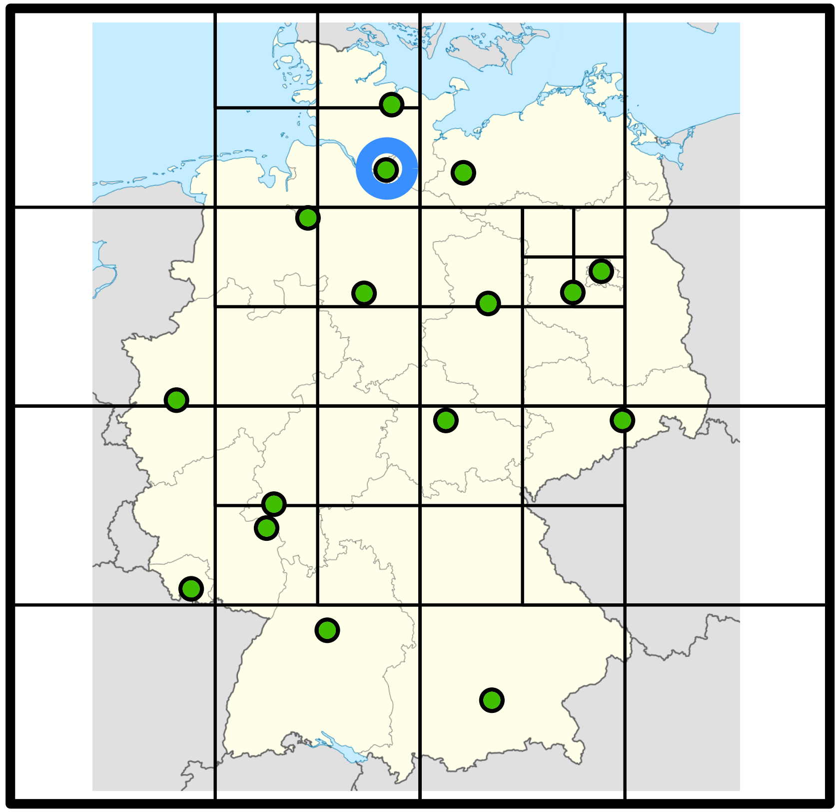

Faster point location:

preprocessing: build hash table using $id(v)$

query: binary search on levels

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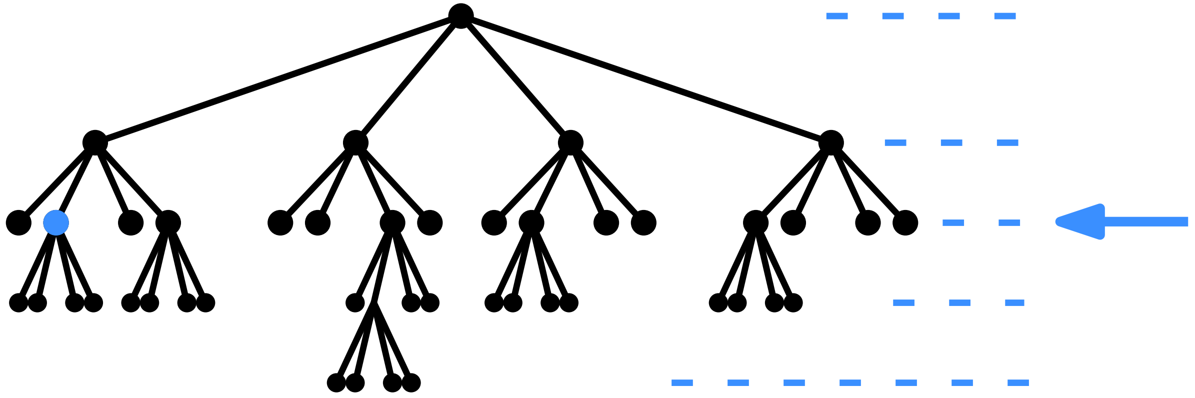

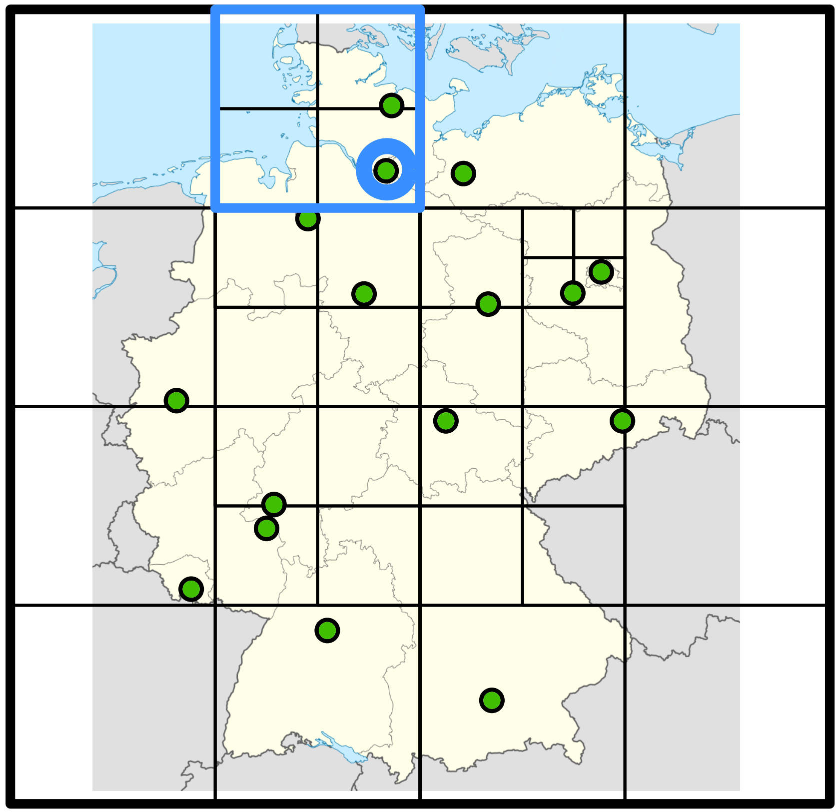

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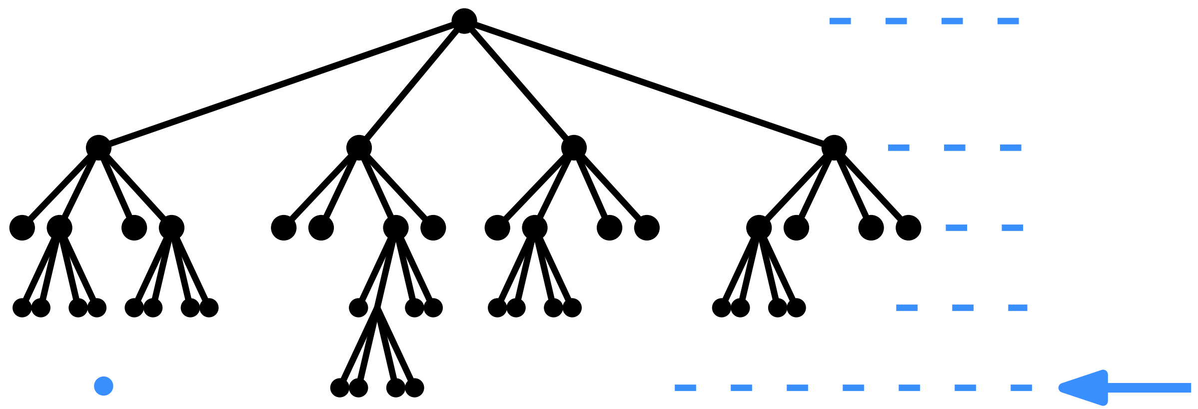

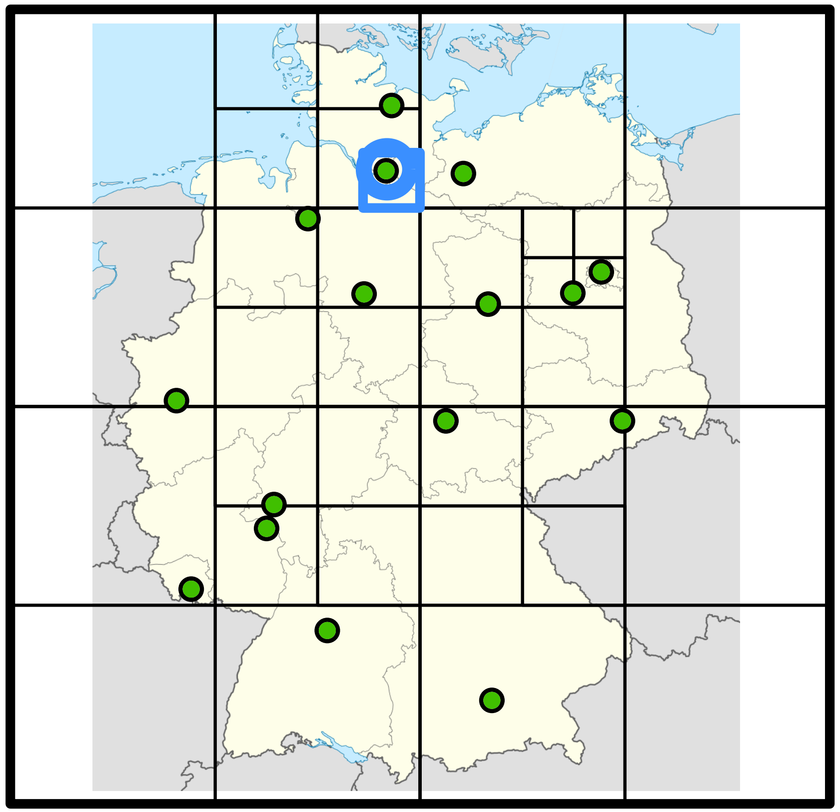

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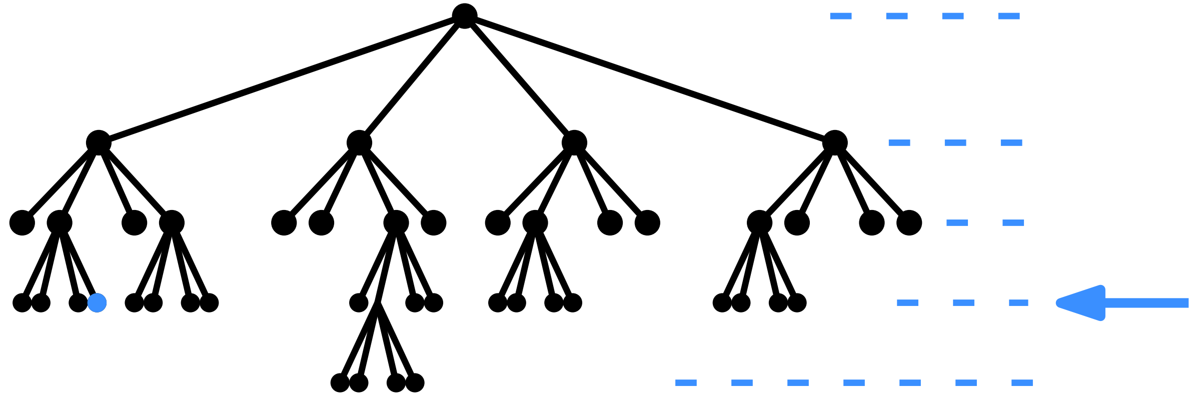

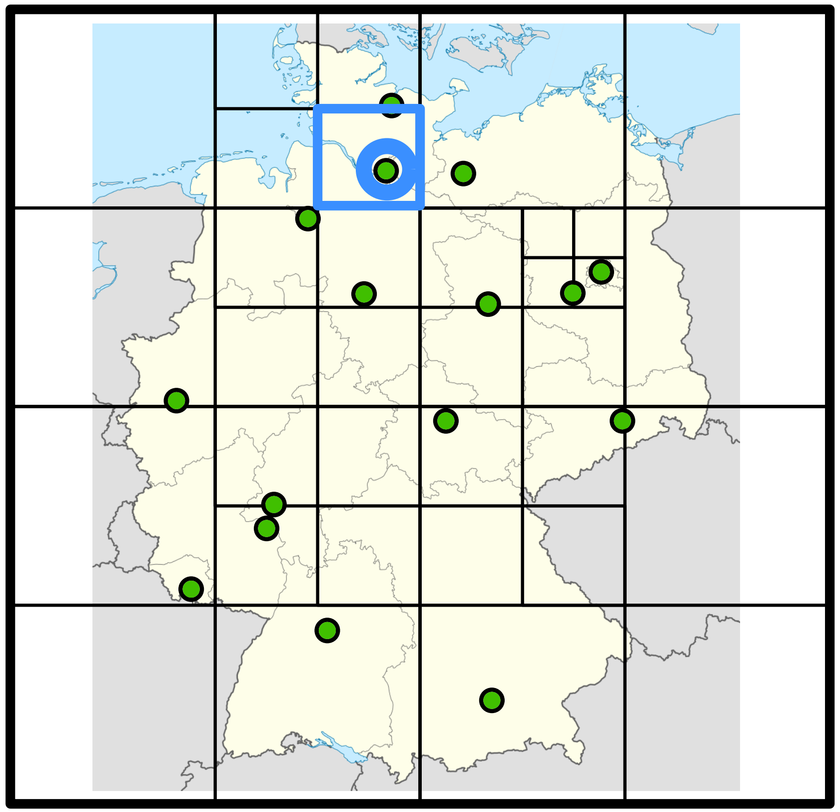

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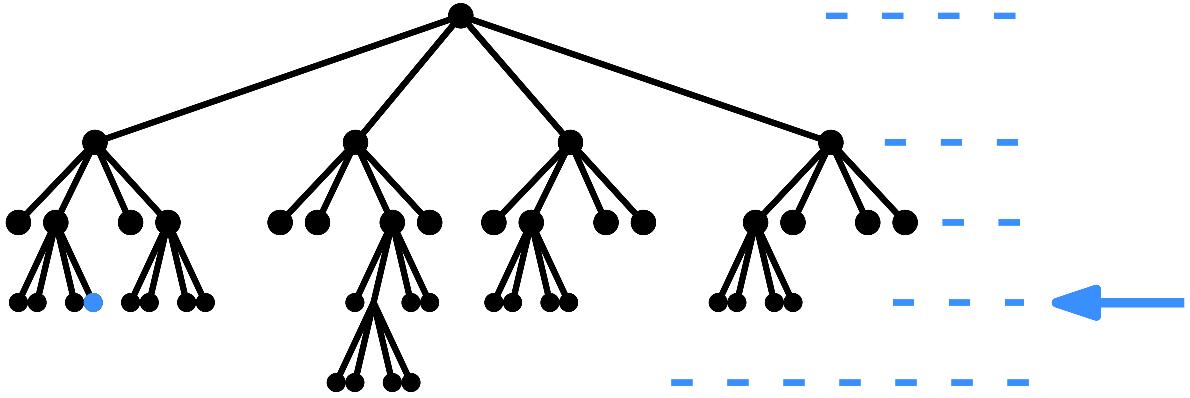

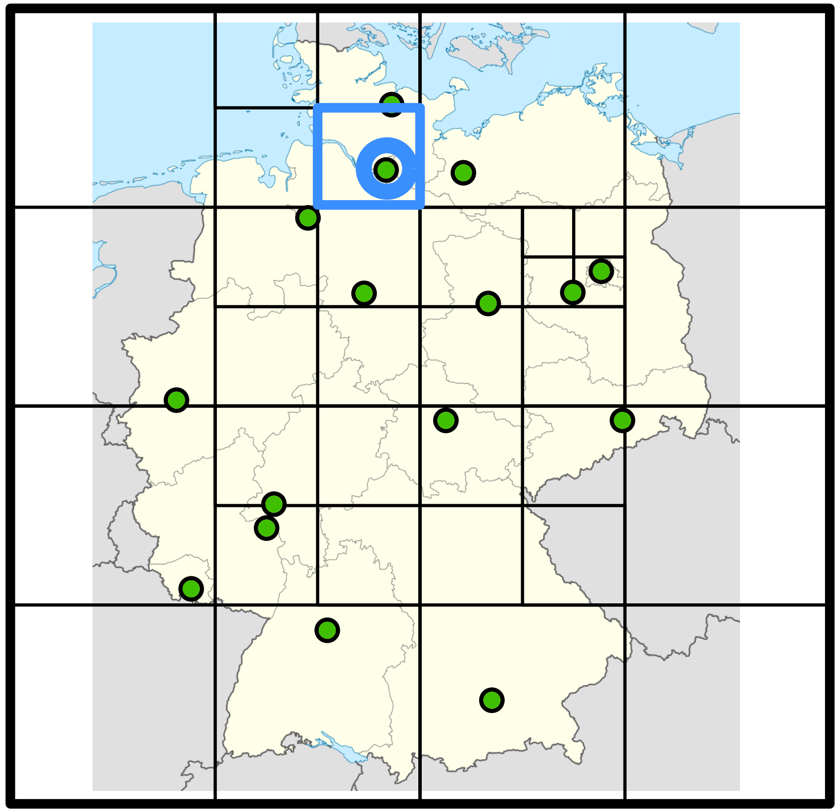

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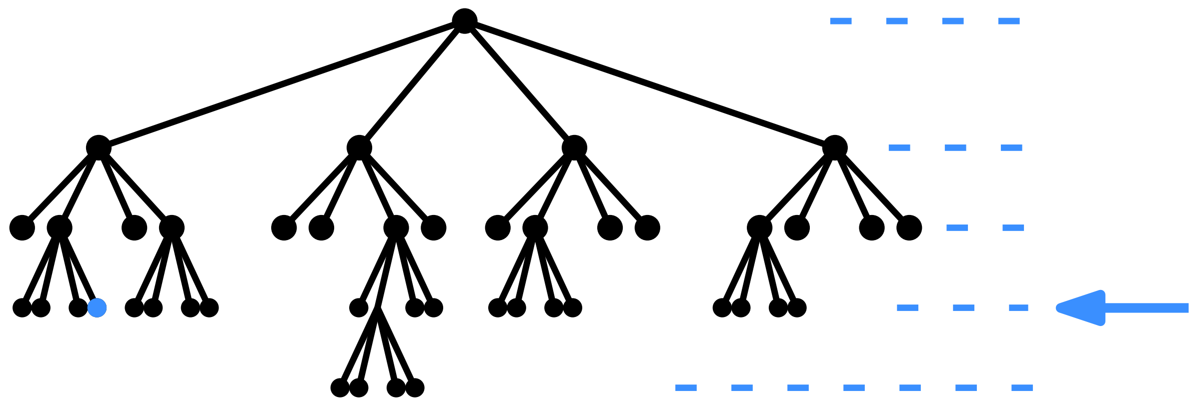

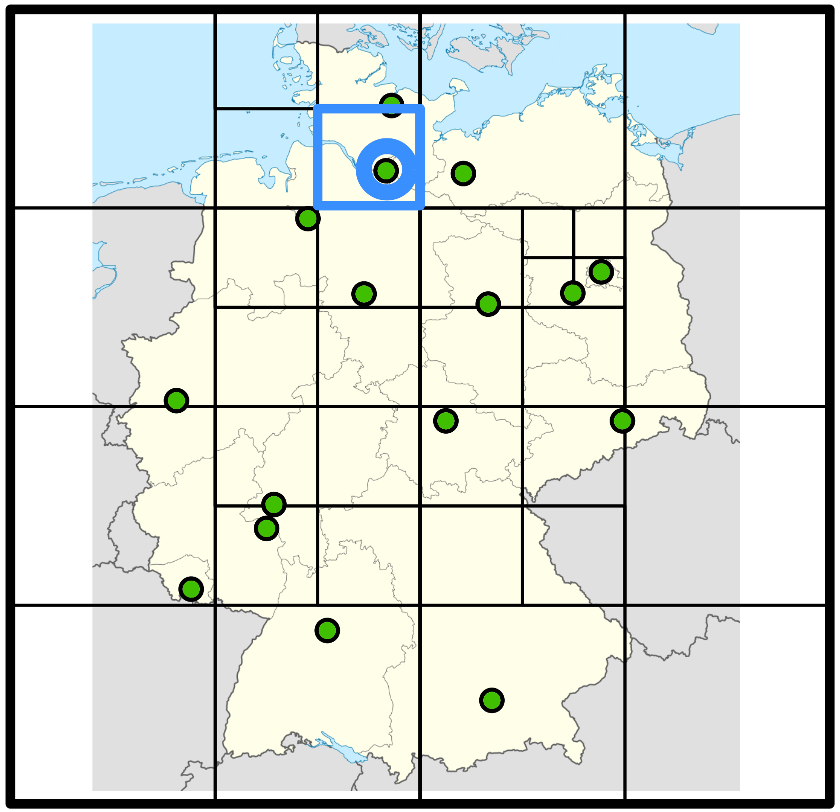

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How large is d? How large is the quadtree?

Quadtree: depth and size

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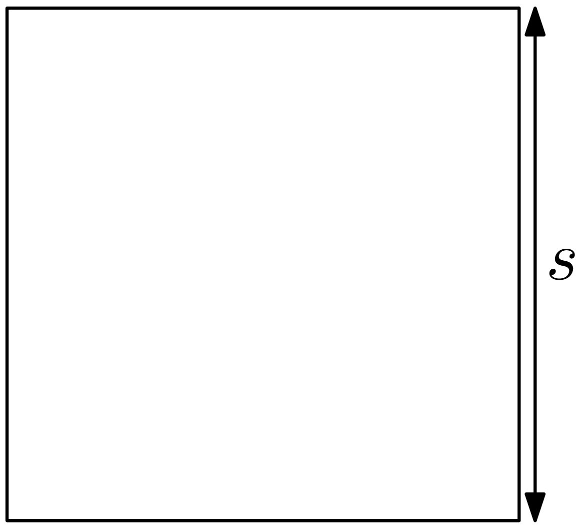

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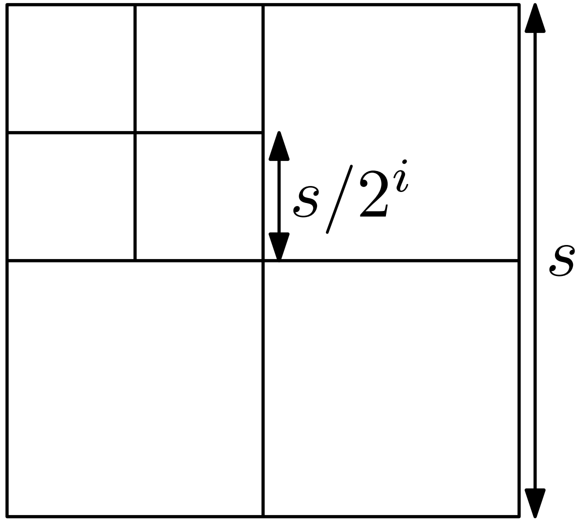

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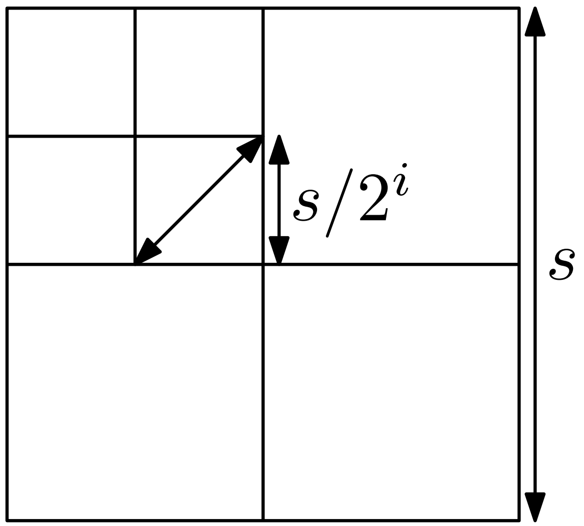

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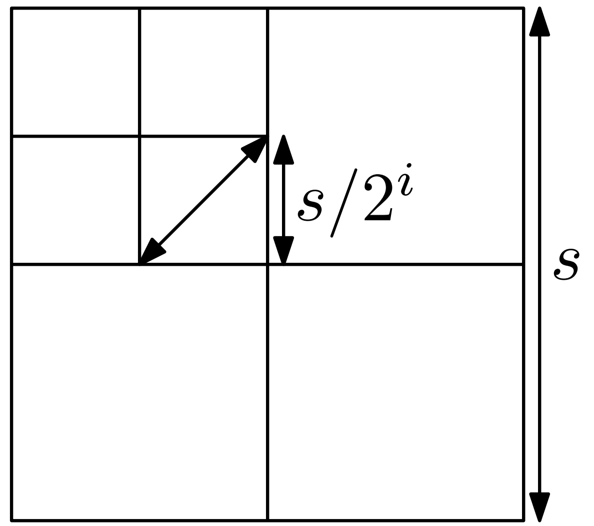

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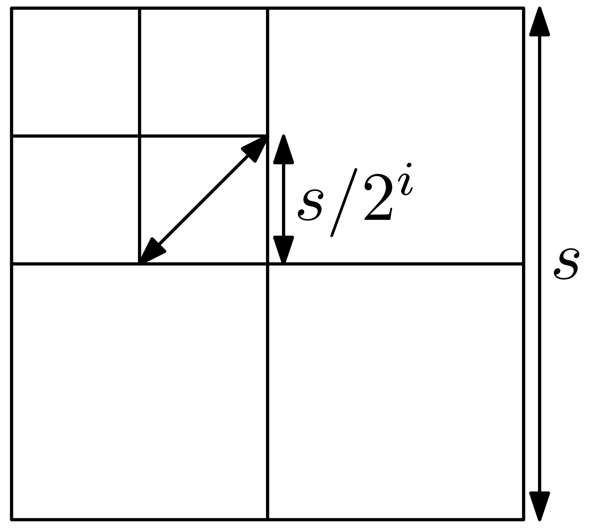
$$\Rightarrow i \leq \log(\sqrt{2}s/c) = \log(s/c) + 1/2$$



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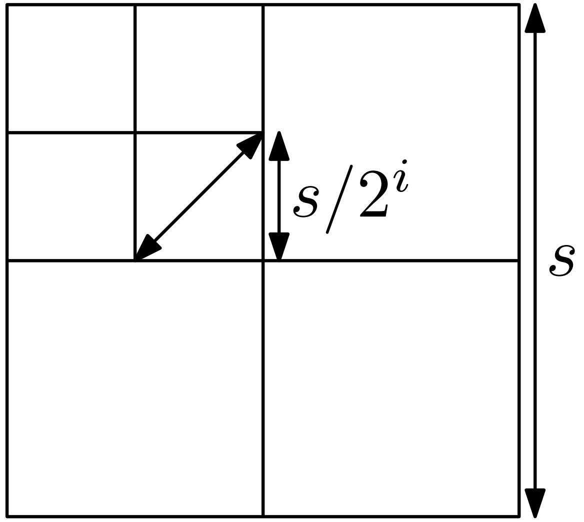

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$\Rightarrow$ depth of quadtree $\leq \log(s/c) + 1/2 + 1$, since nodes with ≤ 1 points have no children

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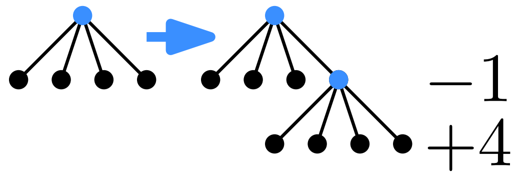
- Inner nodes have 4 children $\Rightarrow$ #leaves $= 1 + 3\cdot$#inner nodes



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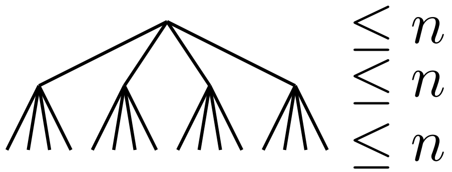

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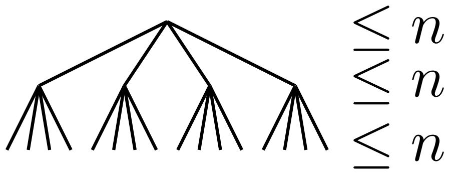

$\Rightarrow$ for depth d overall $O((d+1)n)$ nodes.

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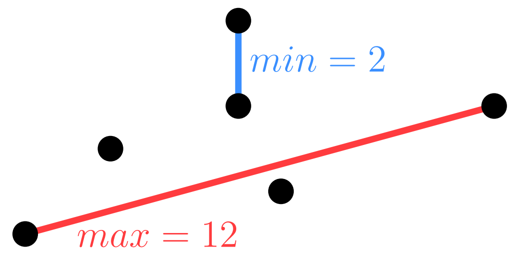
Definition: The spread of point set P is $\Phi(P) = \frac{\max_{p,q \in P} ||p-q||}{\min_{p,q \in P, p \neq q} ||p-q||}$



$$\Phi = \frac{max}{min} = \frac{12}{2} = 6$$

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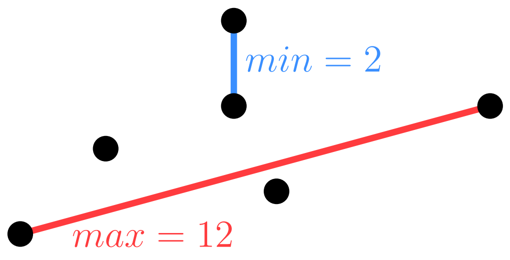

$$\Phi = \frac{max}{min} = \frac{12}{2} = 6$$

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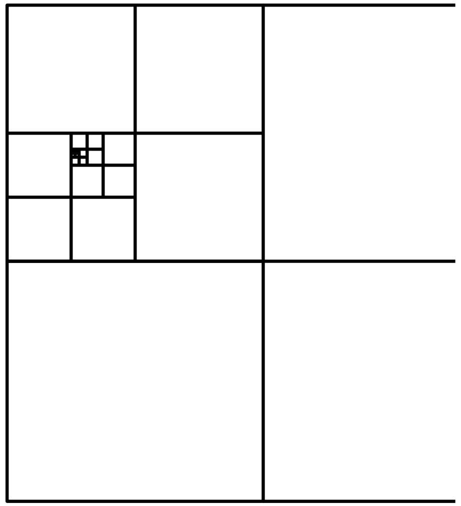

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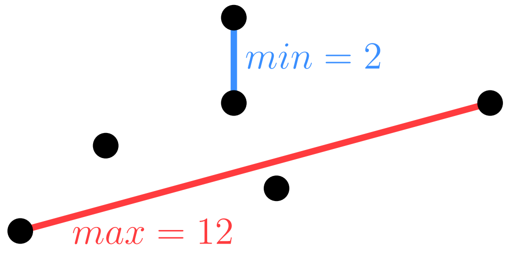

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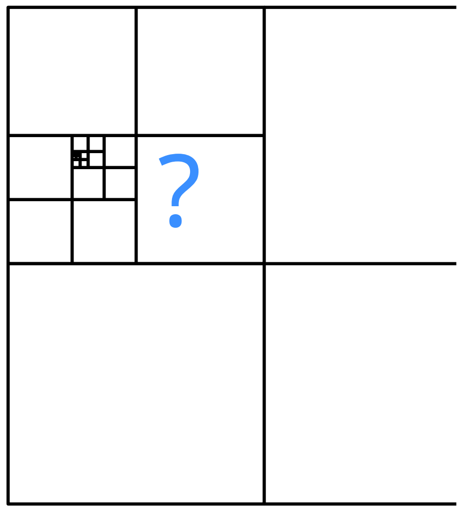
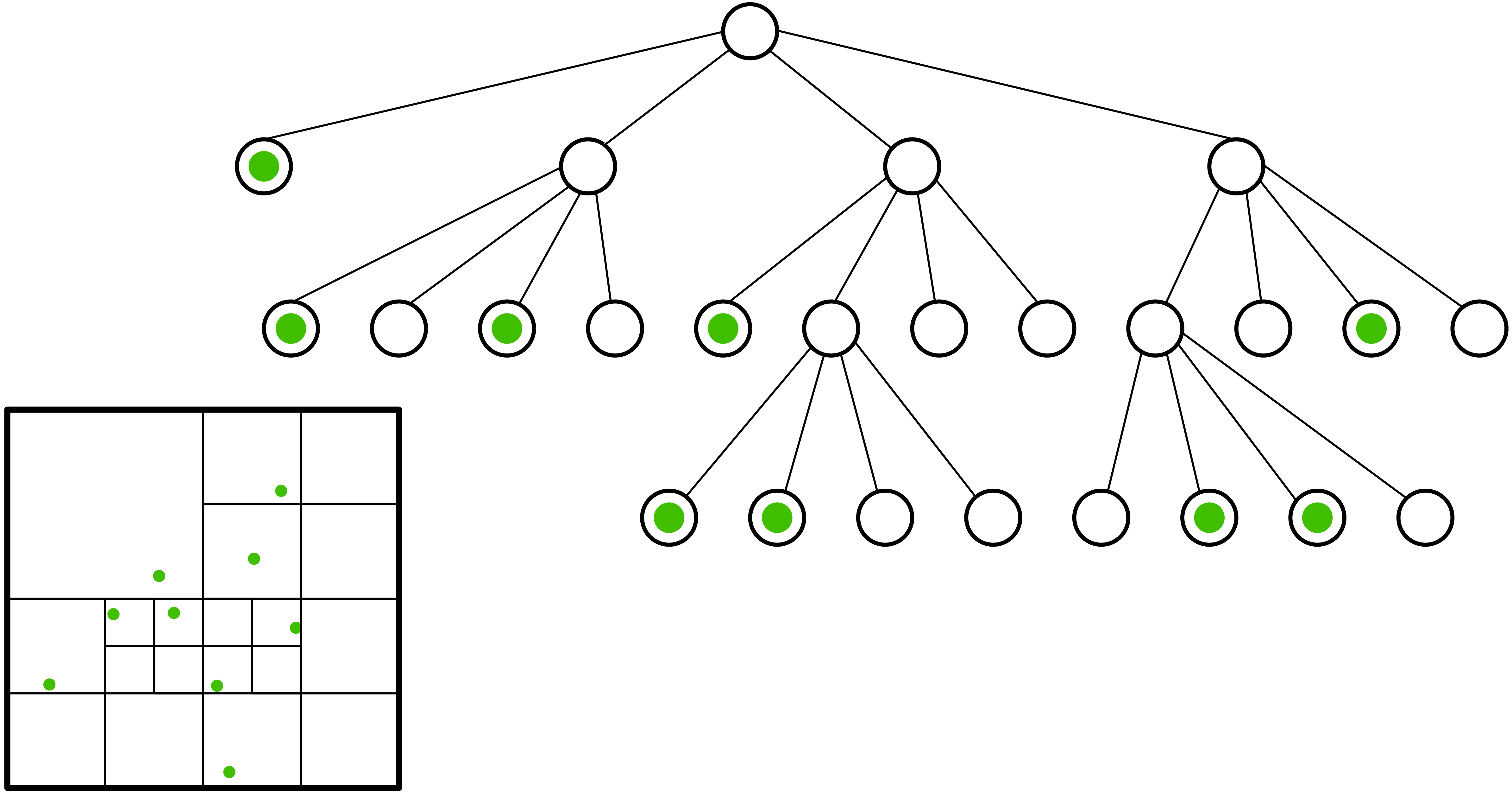
How can we handle the case when $\Phi(P)$ is not bounded by a polynomial in n?   Can we get a linear-size data structure?

Compressed Quadtrees



Improving the size, step 0



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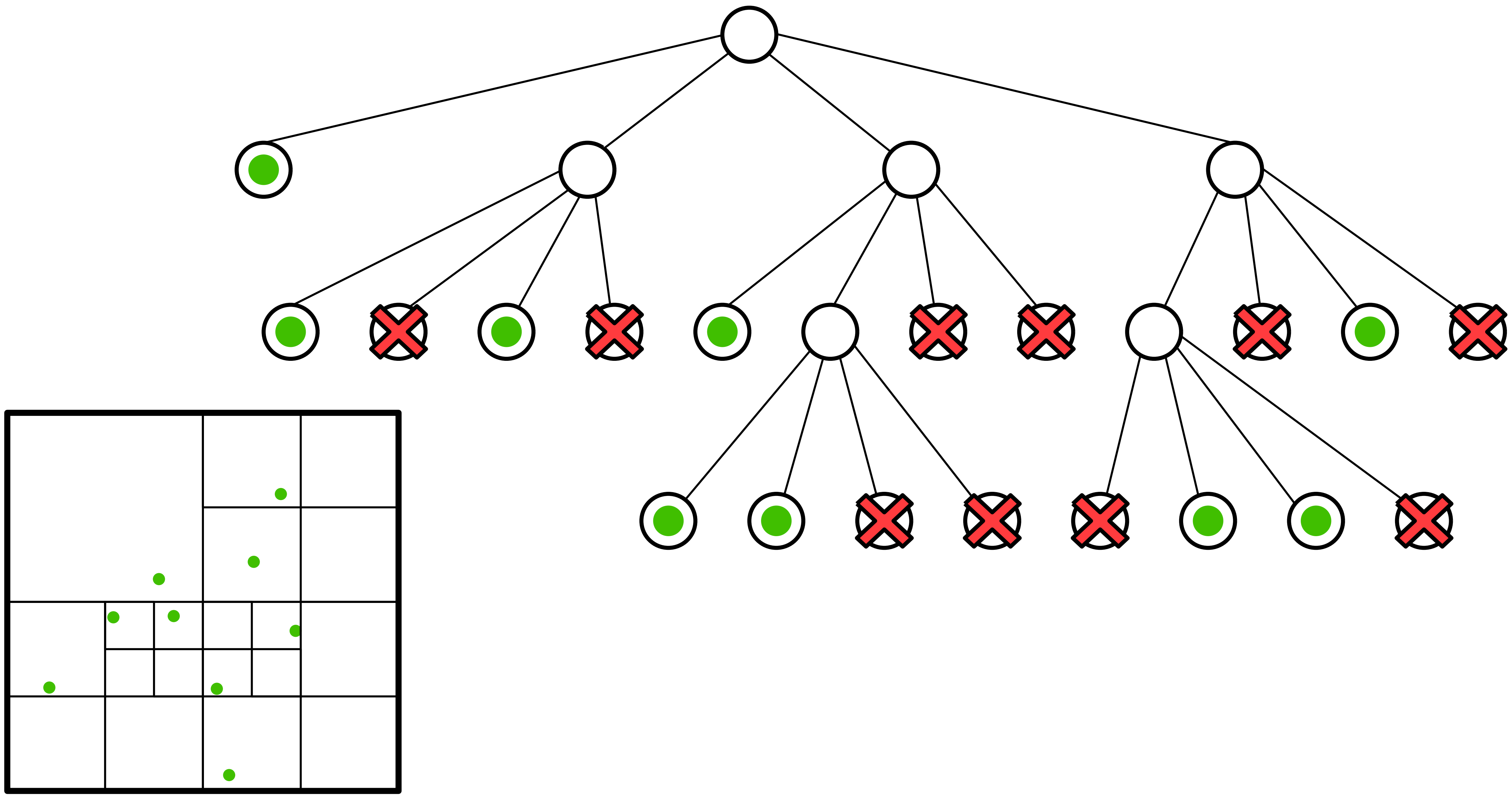

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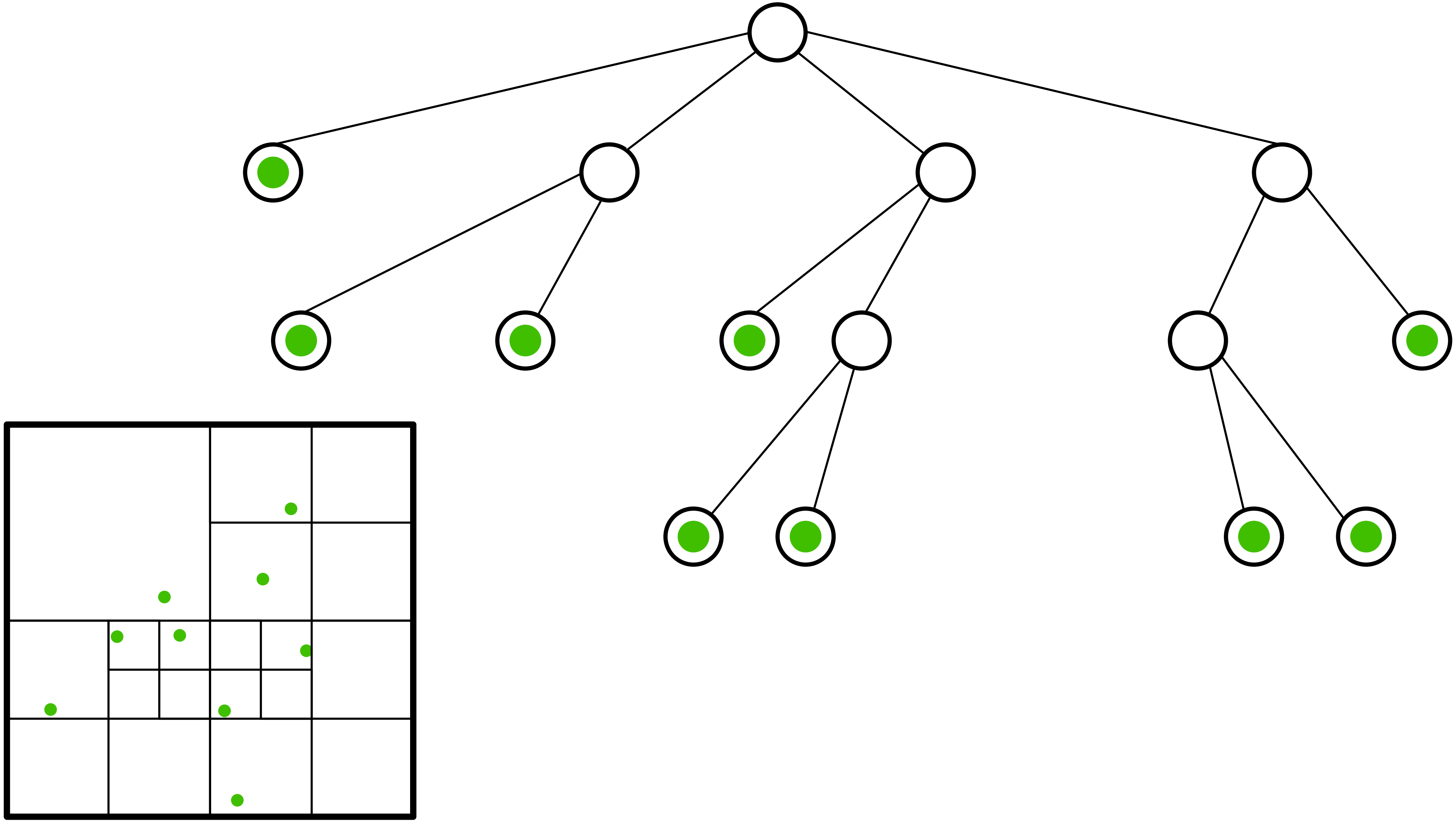

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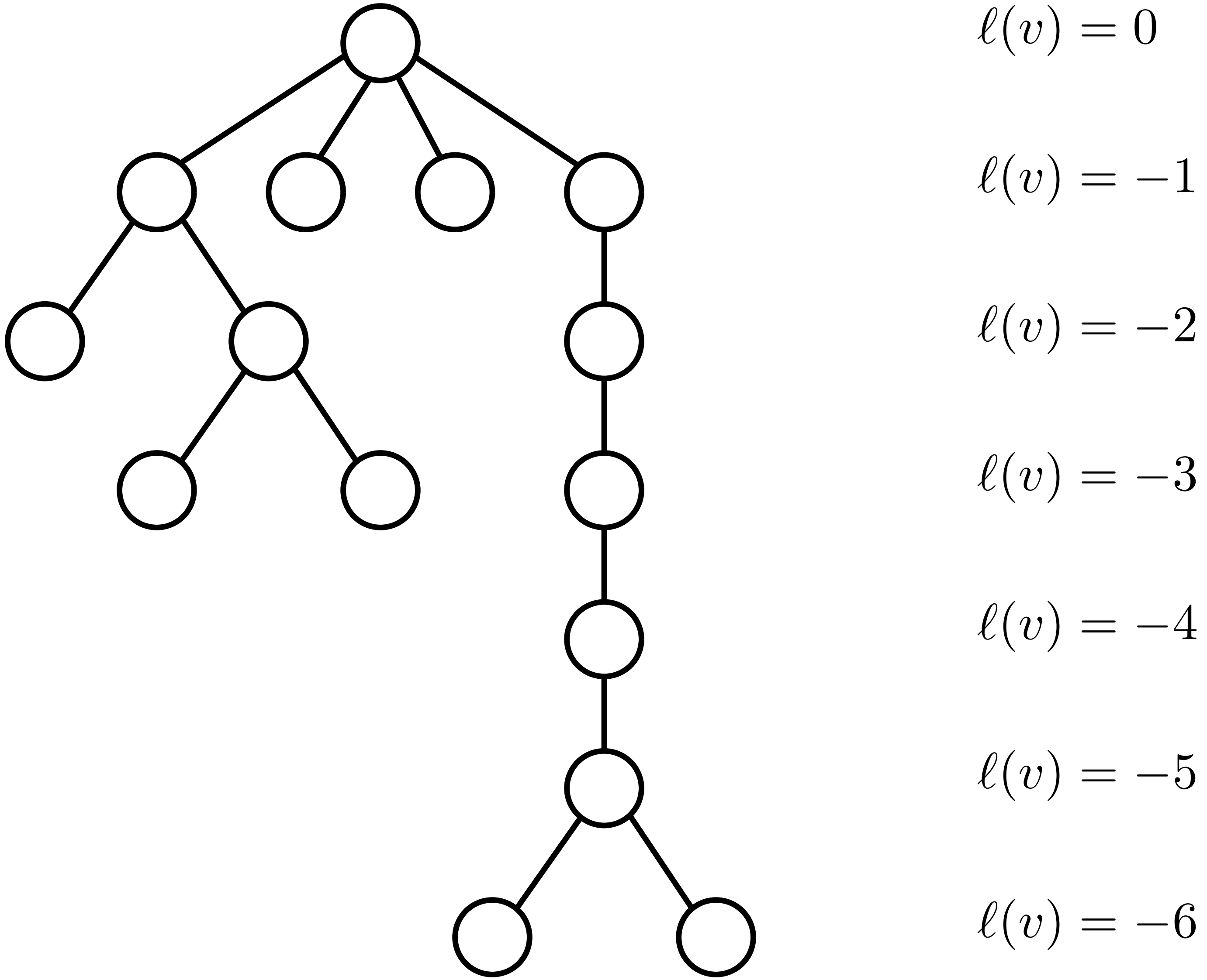
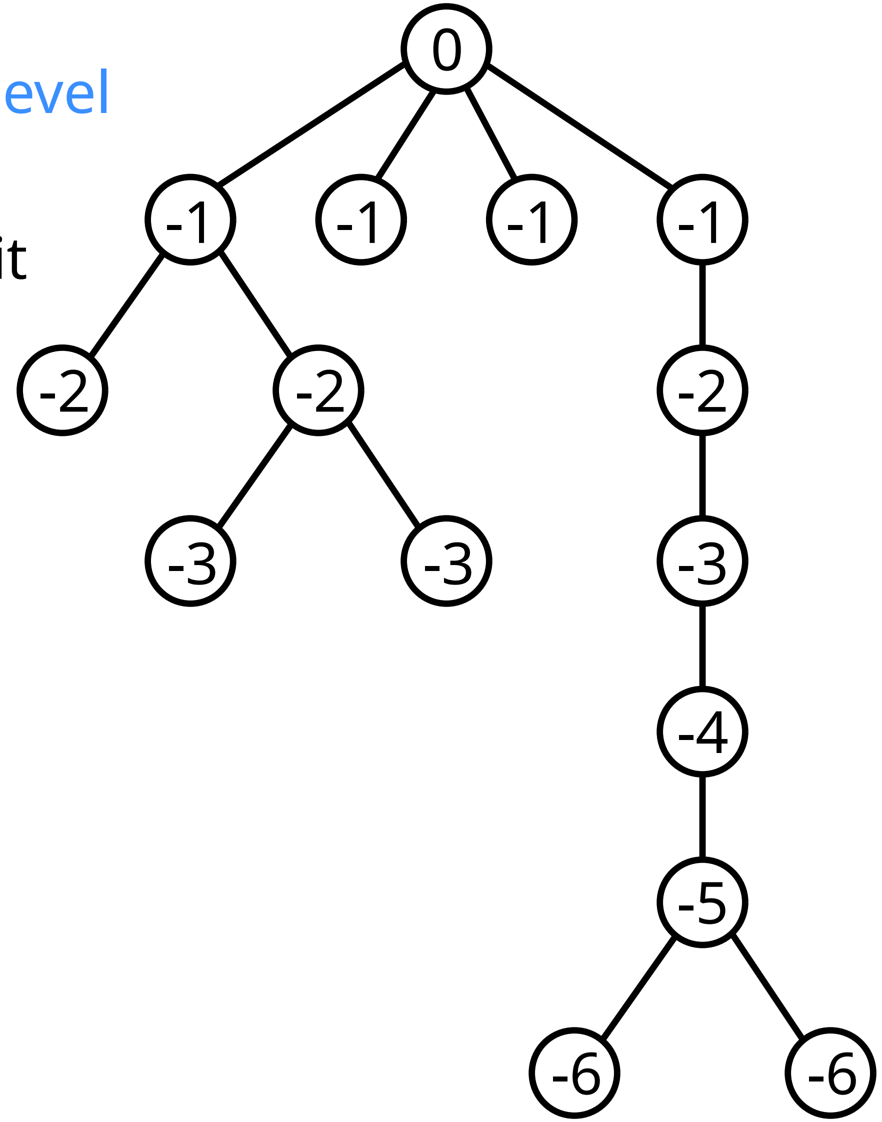


Compressed Quadtrees

Each node gets:

- An integer denoting its level in the original quadtree
- A pointer to the square it represents.



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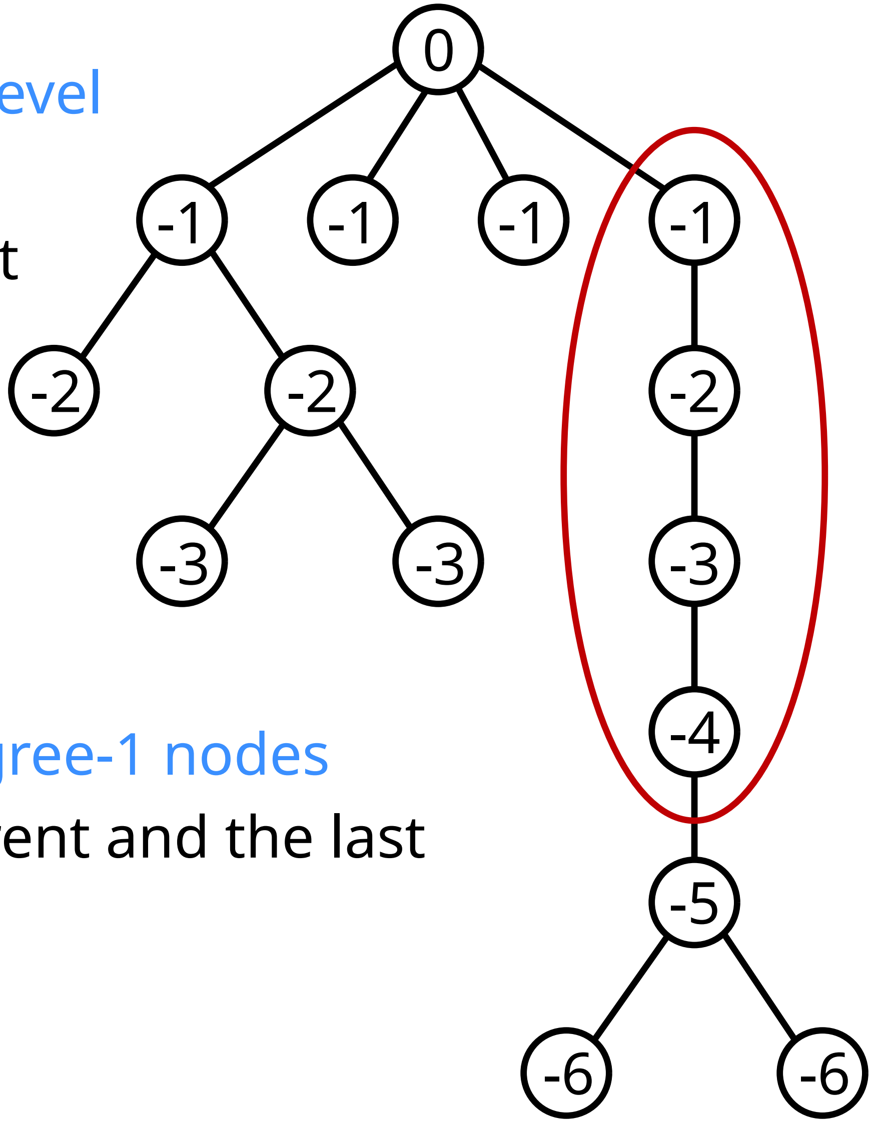

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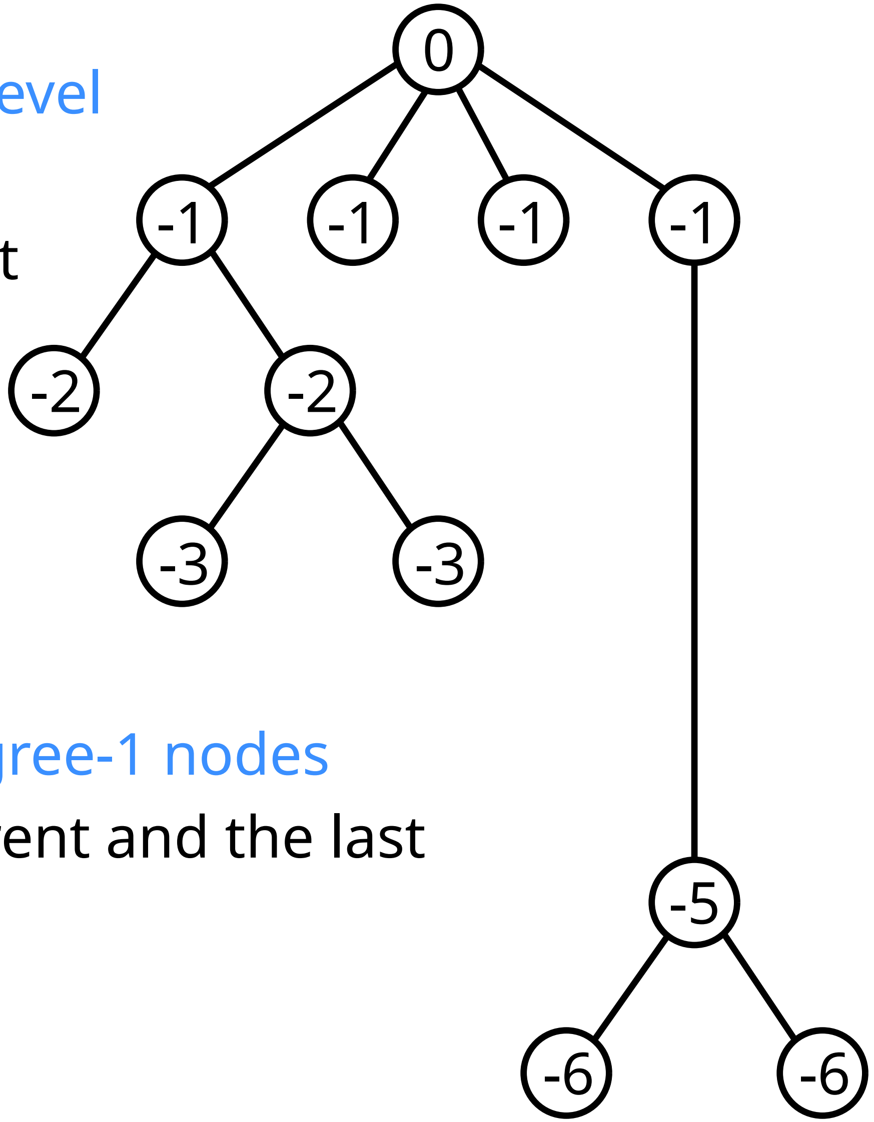

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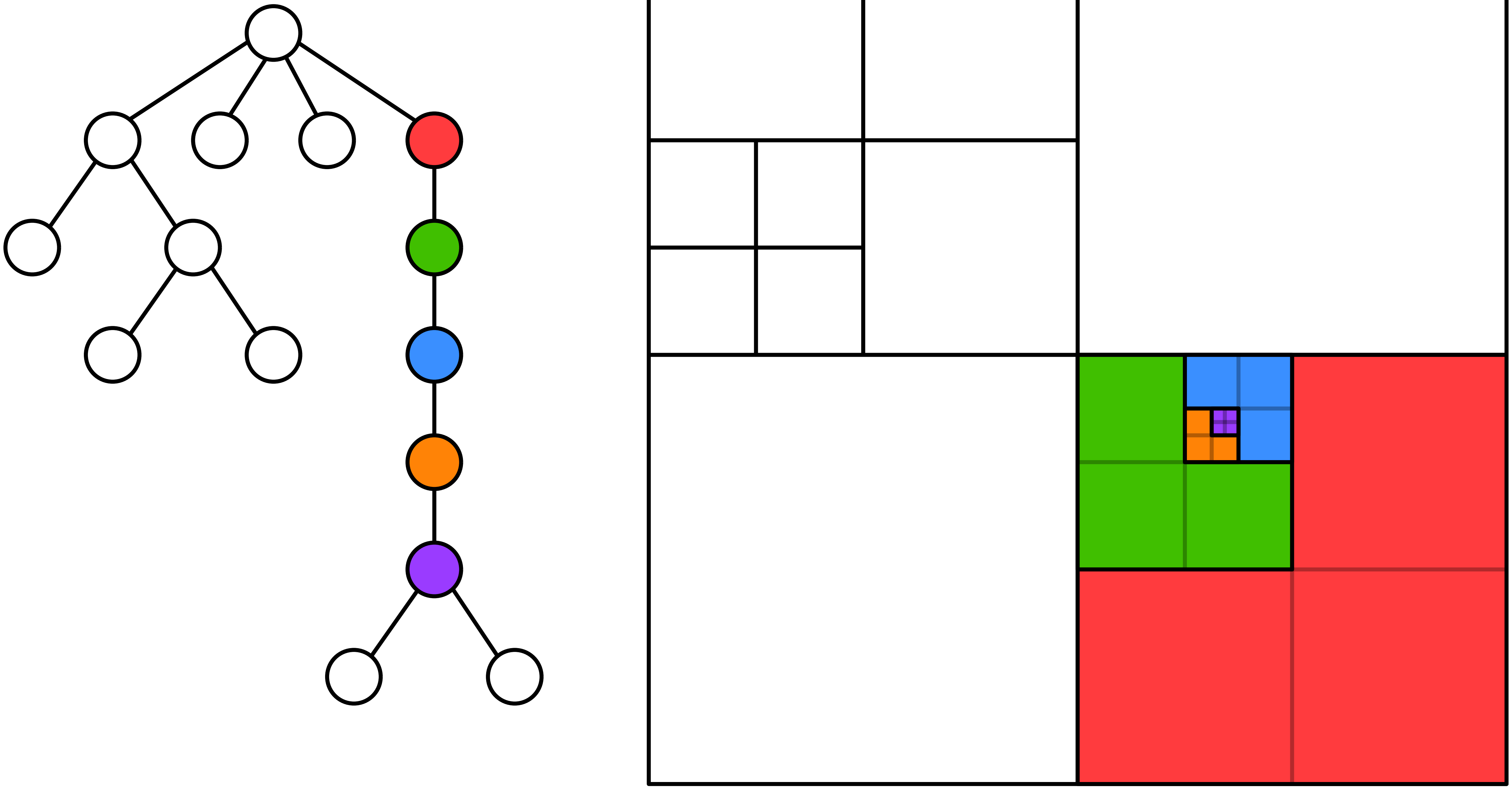

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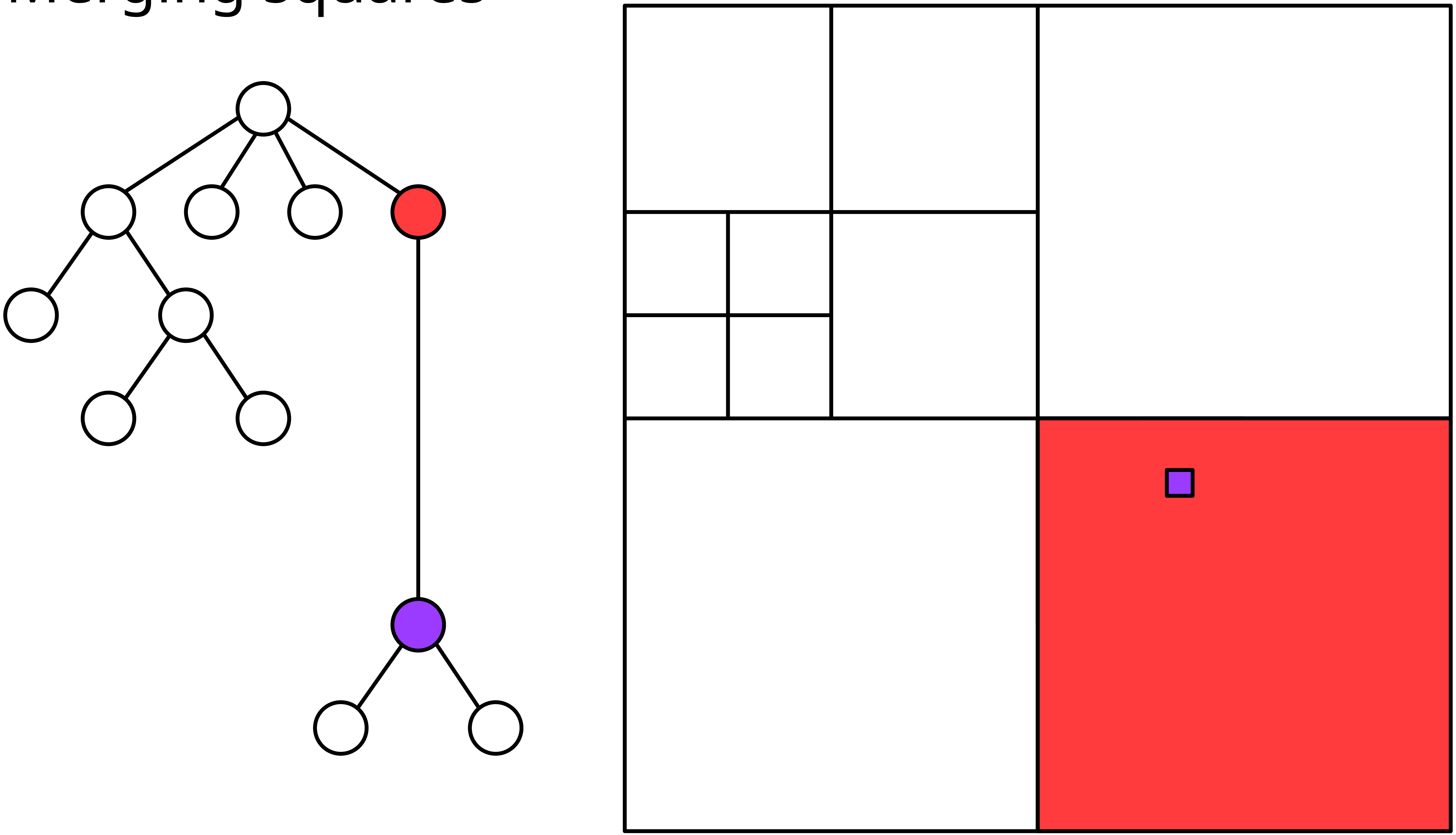
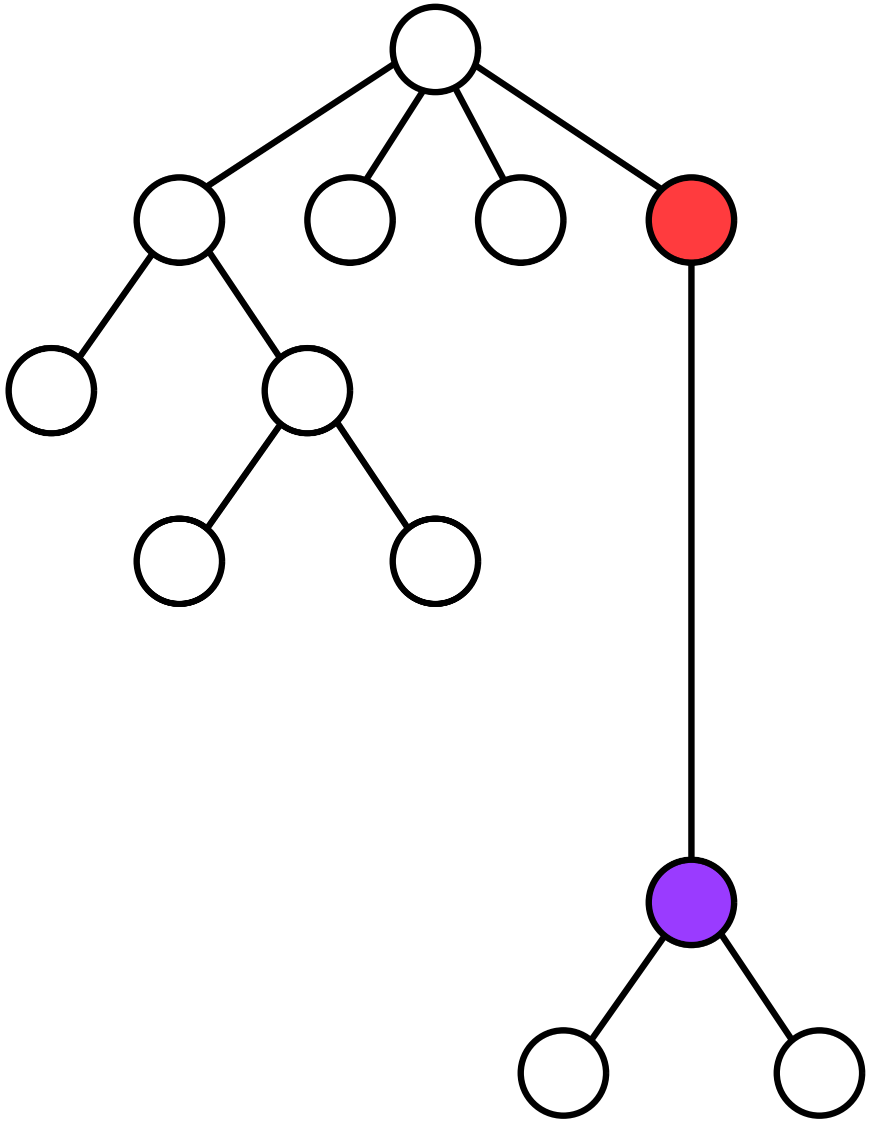
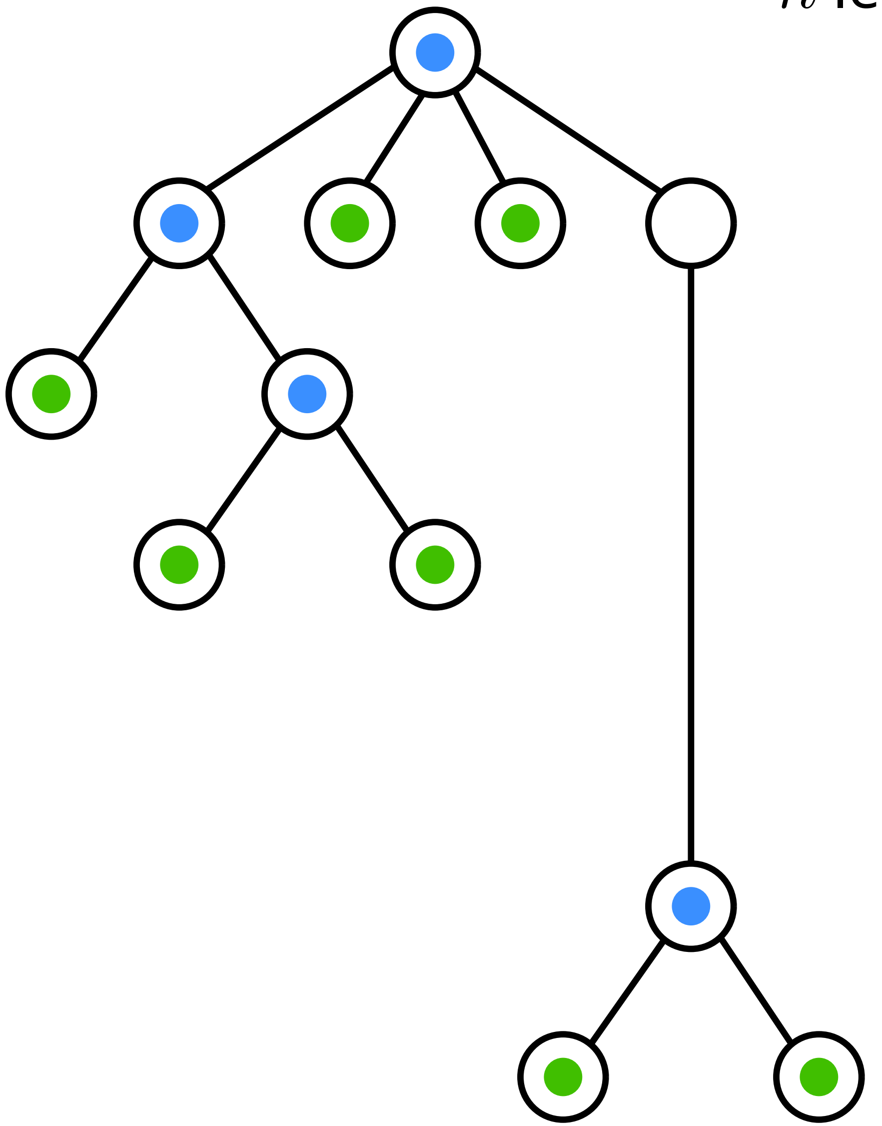




What the size of the compressed quadtree?

Size of compressed quadtree

n leaves $\Rightarrow$ at most $n-1$ internal nodes with degree ≥ 2



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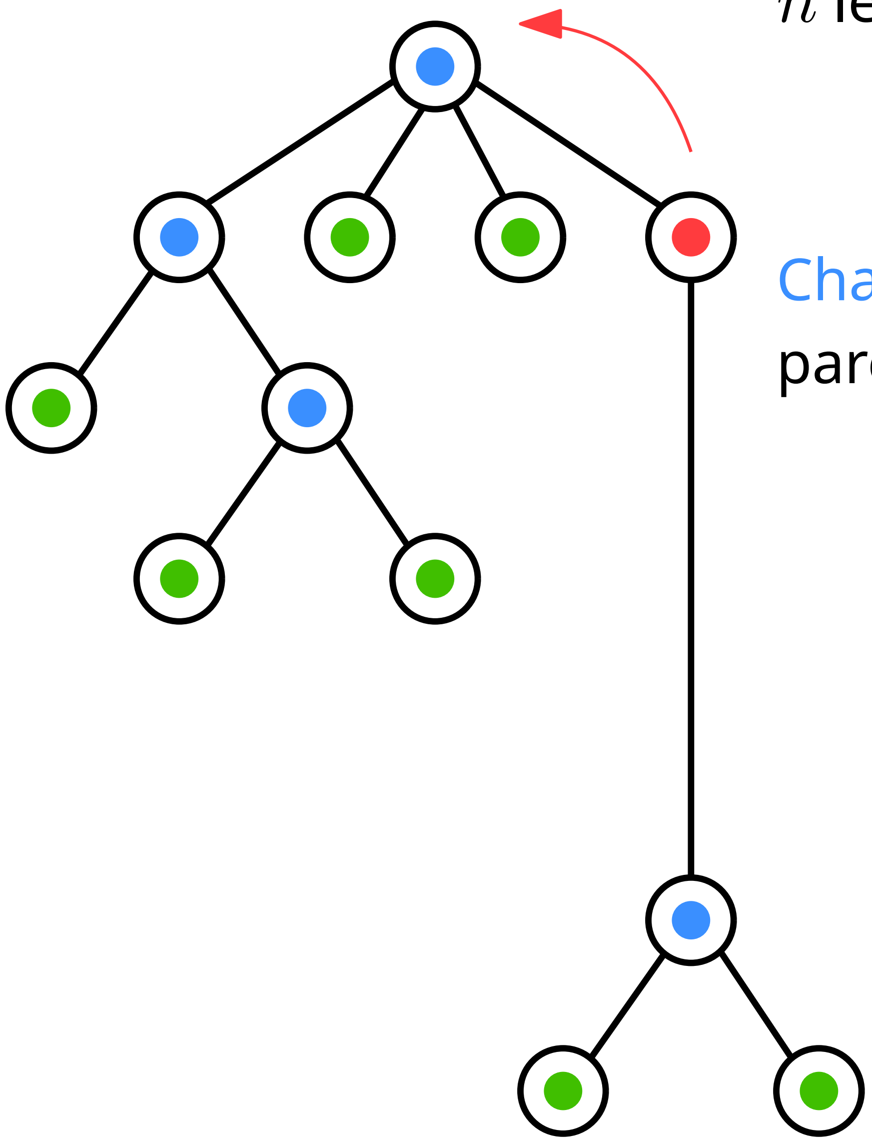

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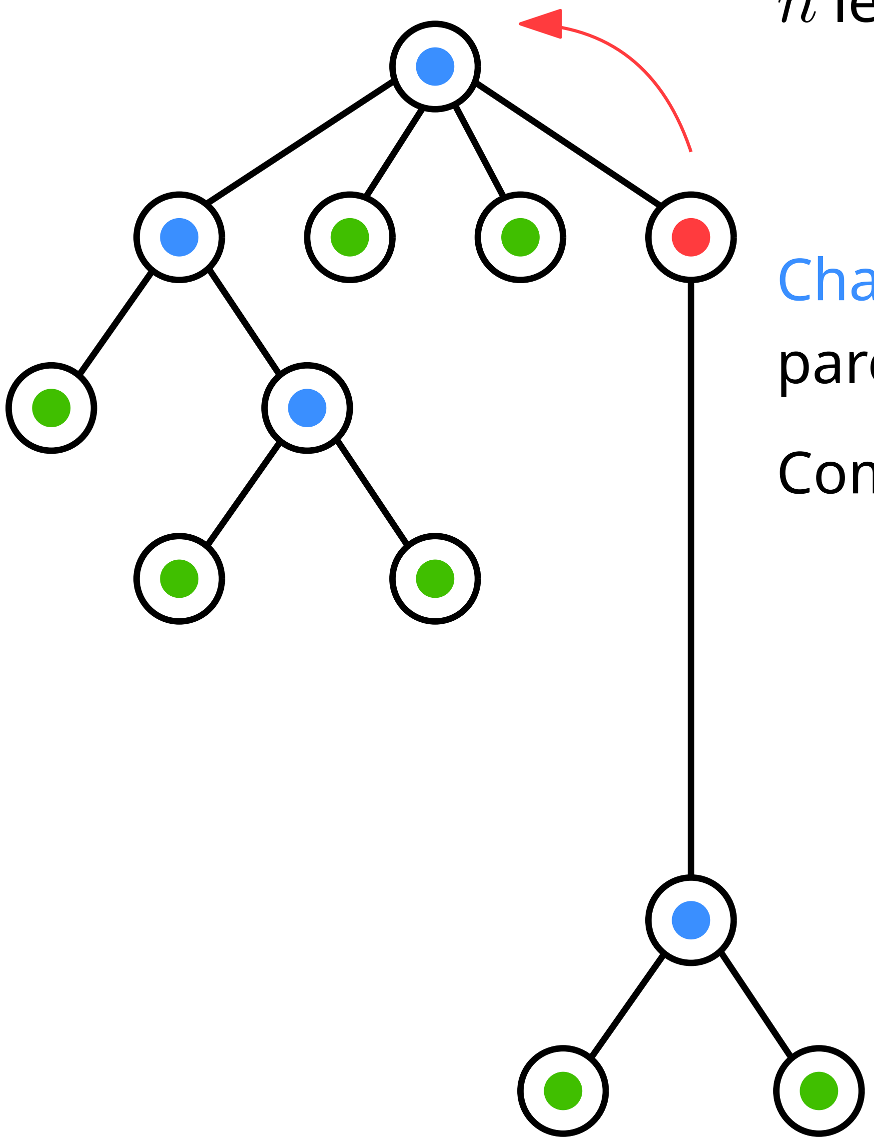

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Compressed quadtrees have linear size!

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Simple recursive construction on compressed quadtrees has unbounded time complexity when the spread of the point set is unbounded.

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Theorem: In linear time we can compute a disk D containing $n/10$ points with radius $r_D \leq 2r_{OPT}$, where r_{OPT} is the radius of the smallest disk containing $n/10$ points.

Question: Which algorithm(s) do you know to compute this disk?

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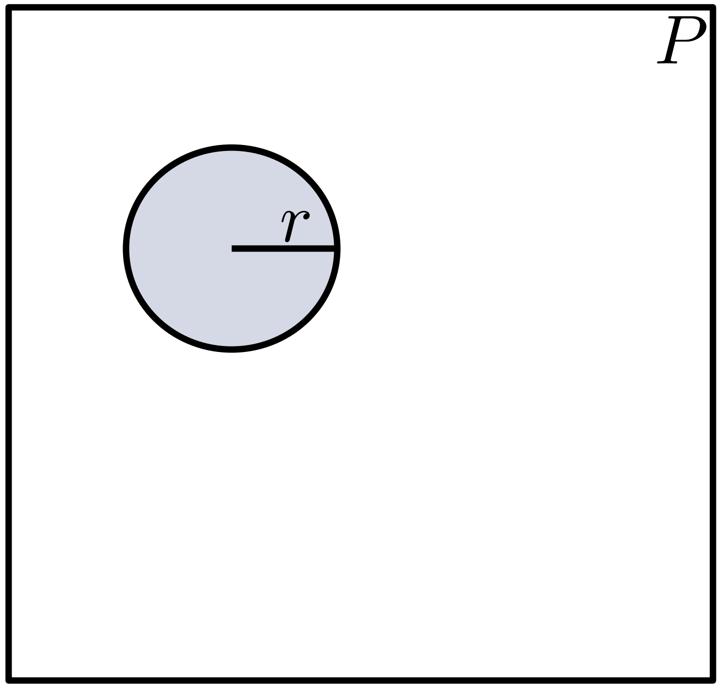

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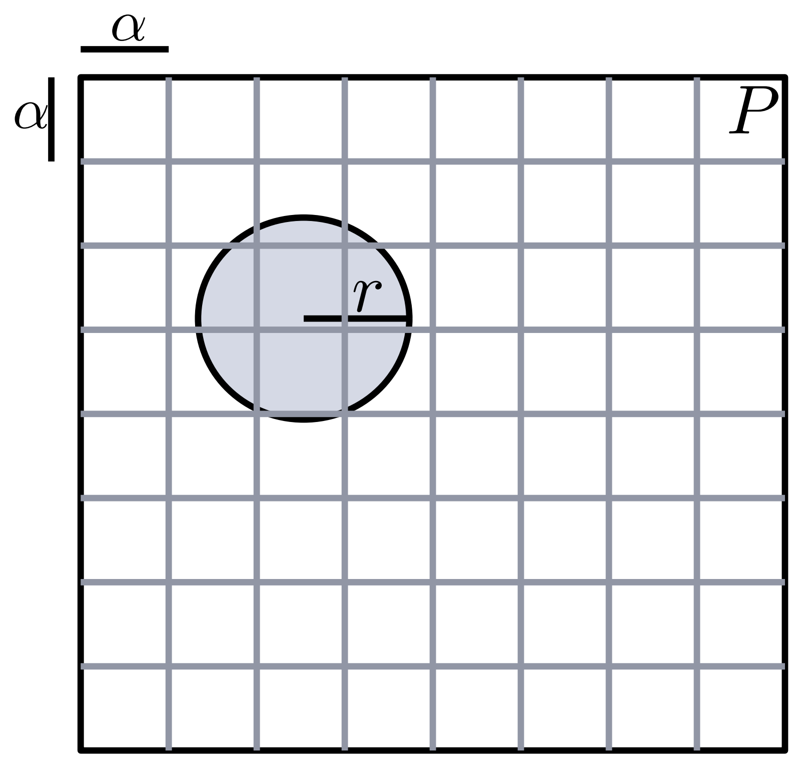

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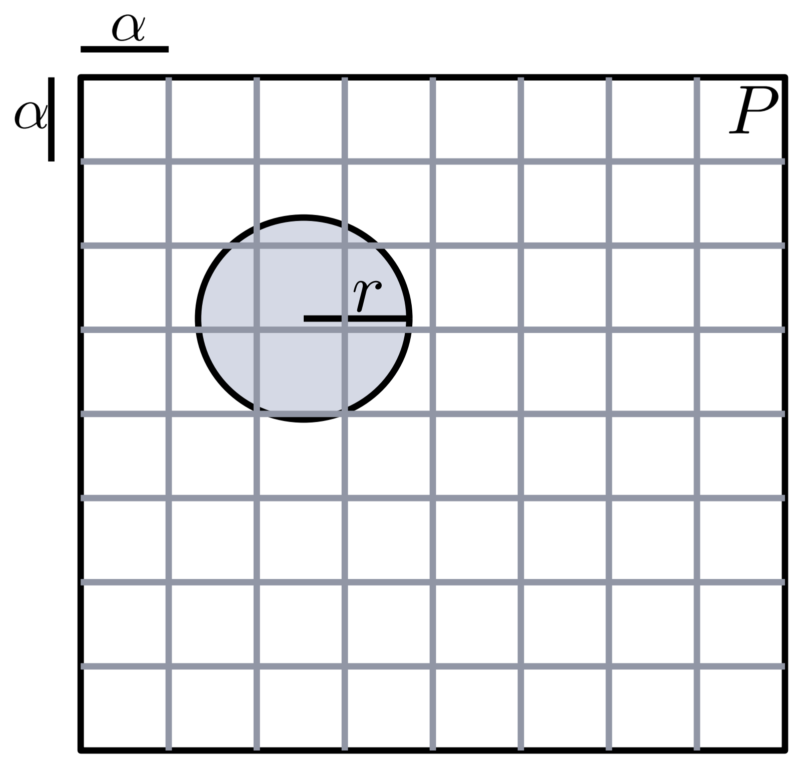
$\implies$ D is covered by at most 25 grid cells



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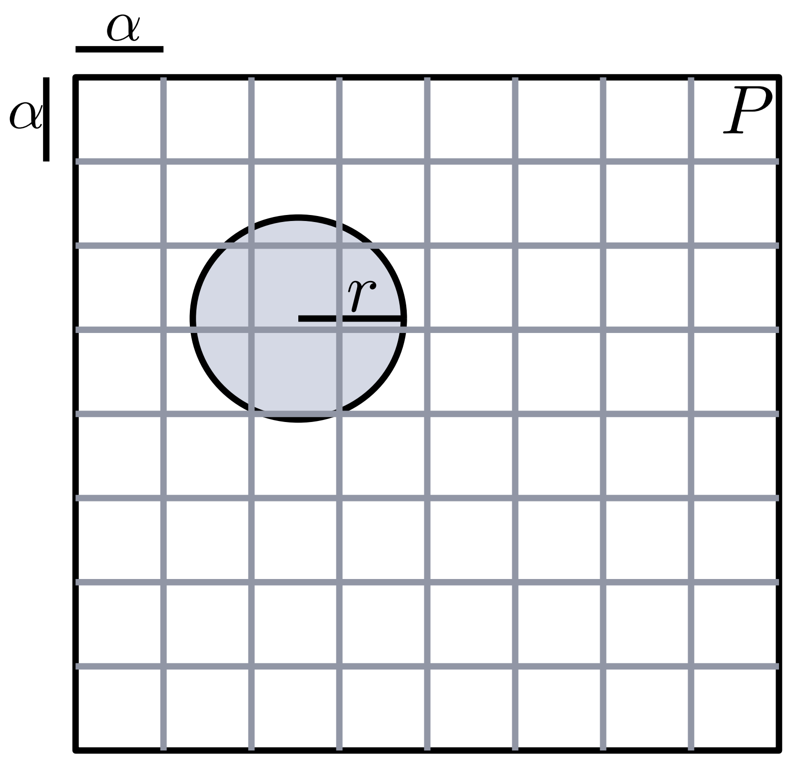

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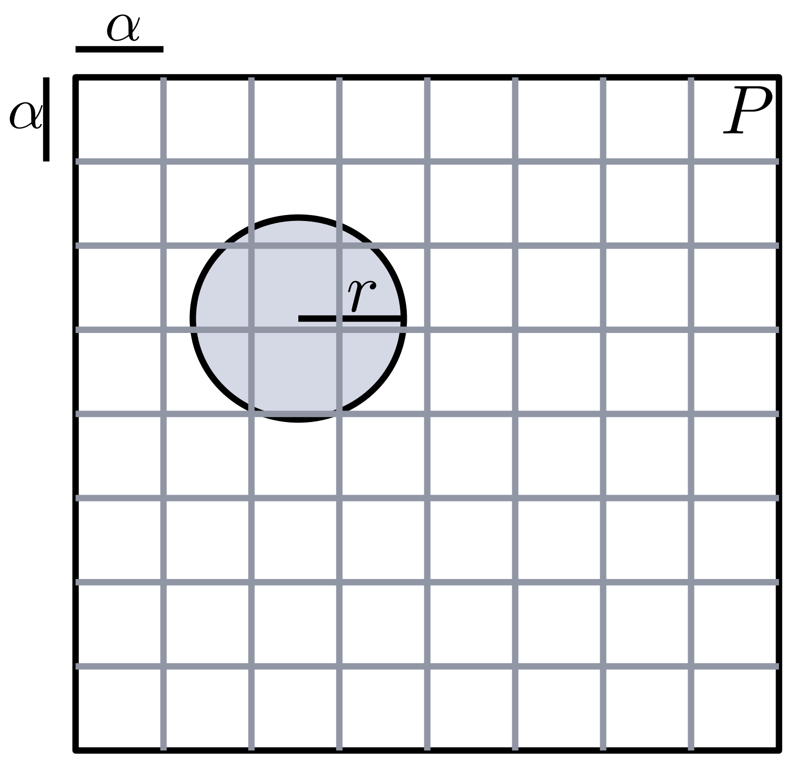

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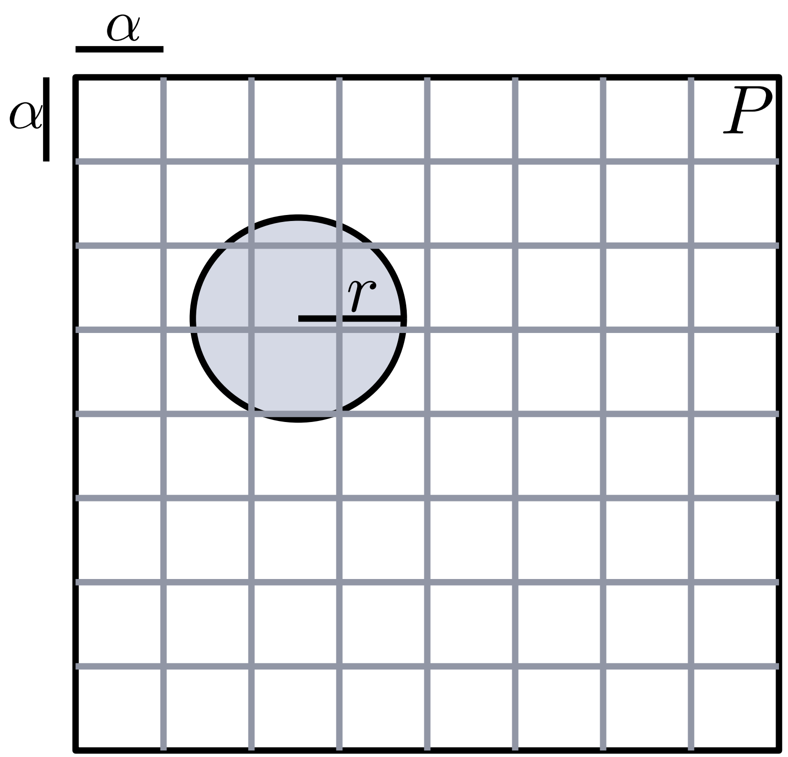
Let $\square$ denote the cell containing the largest number of points.

$$P_{in} = P \cap \square \text{ and } P_{out} = P \setminus P_{in}$$

Note that $|P_{in}| \geq n/250$ and $|P_{out}| \geq n/2$



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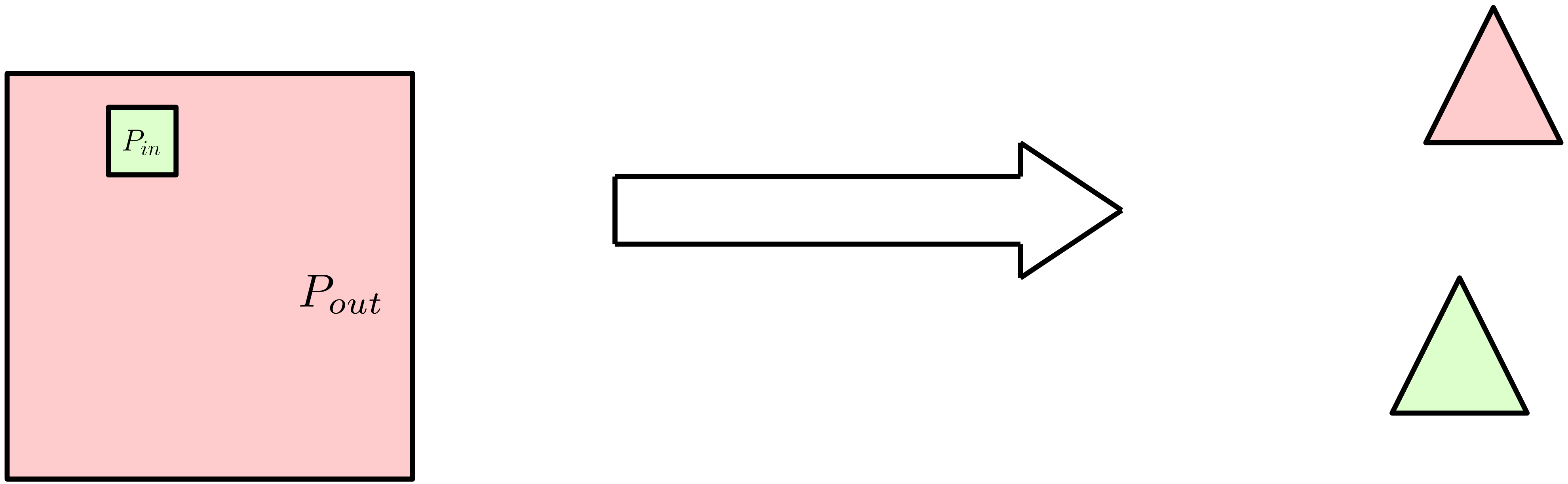
Recursively construct quadtrees for P_{in} and P_{out}



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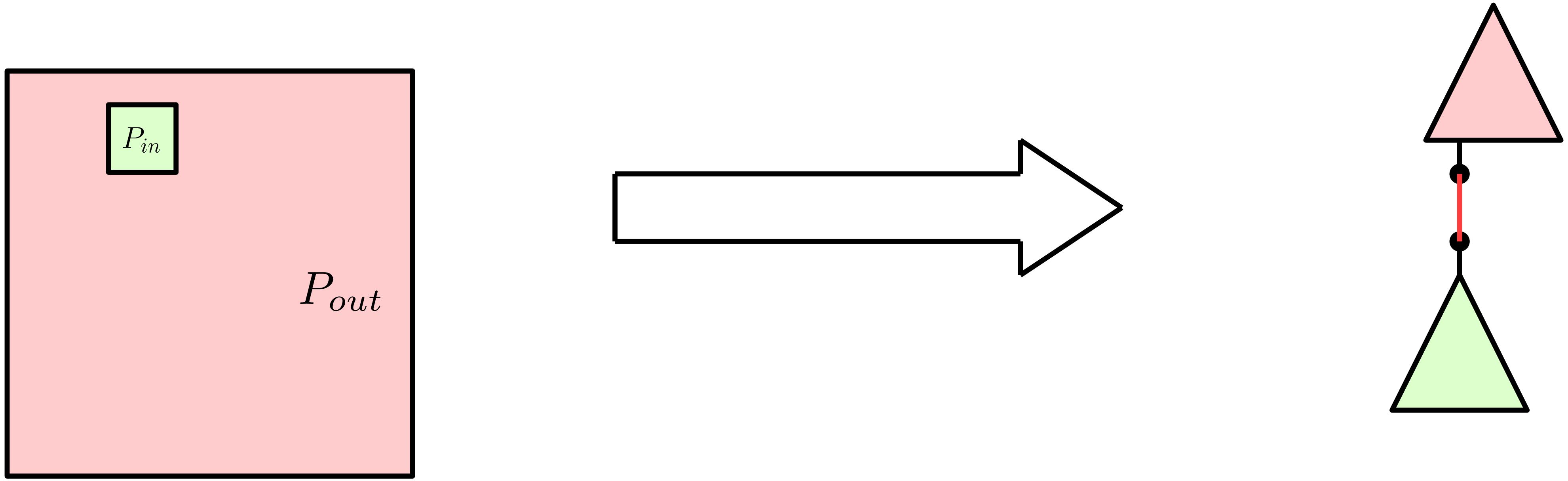
Create a node representing $\square$ in both quadtrees.



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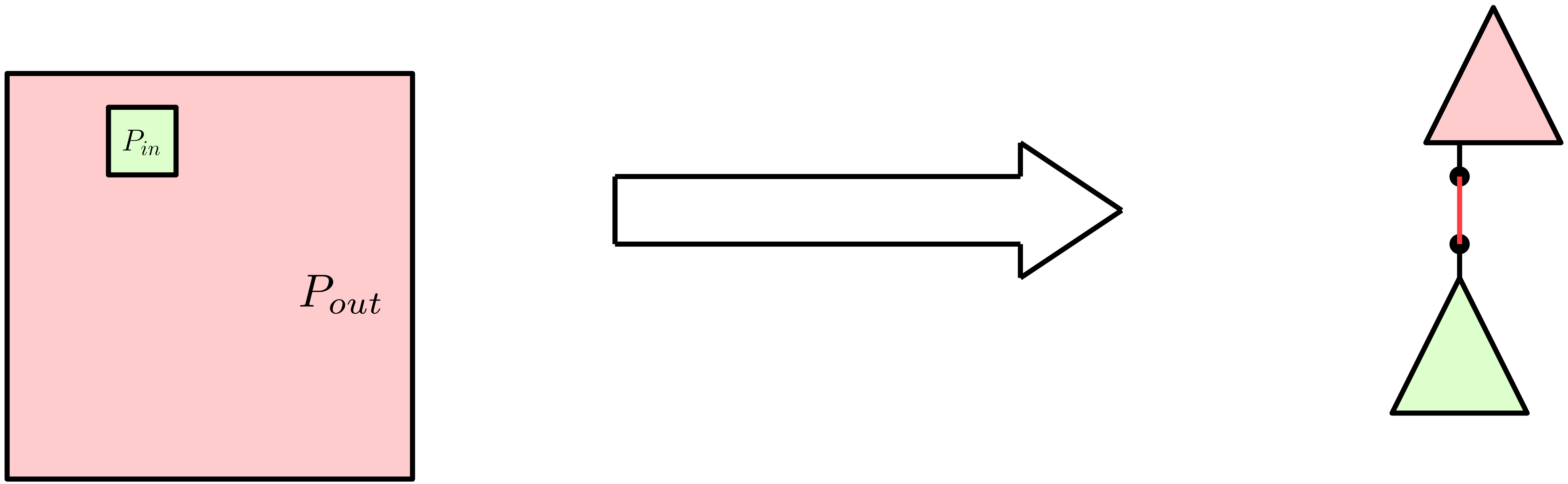

Construction time: $T(n) = O(n) + T(|P_{in}|) + T(|P_{out}|) = O(n \log n)$

Quiz

What is the maximum depth that a quadtree on n points can have?

A $\Theta(\log n)$

B $\Theta(\sqrt{n})$

C $\Theta(n)$

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Question: How does such a quadtree look like?

Point-location on compressed quadtrees

Given a compressed quadtree T of size n, find lowest node in the tree containing point q.

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Alternative: preprocess T into a balanced tree T' with cross-pointers to T.

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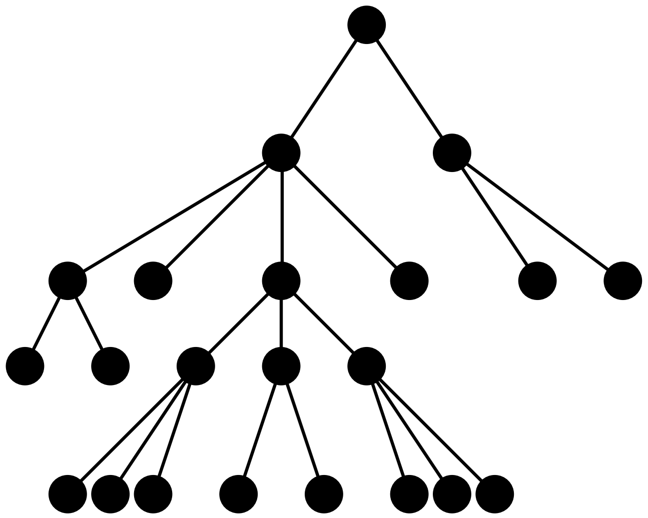

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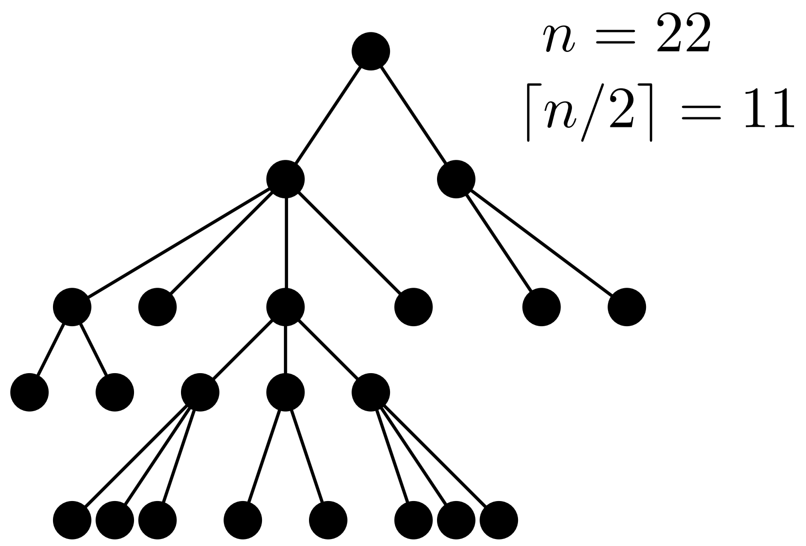
Walk through the tree starting at root, going into the subtree that contains $\geq \lceil n/2 \rceil$ nodes.



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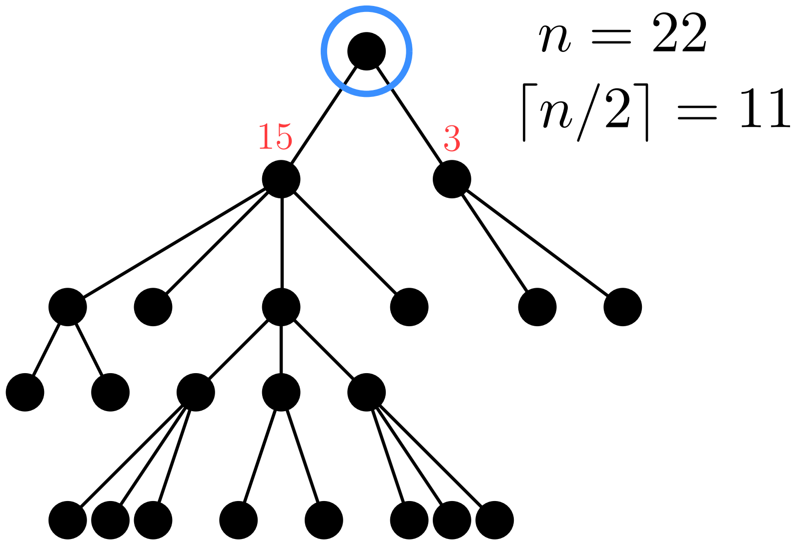

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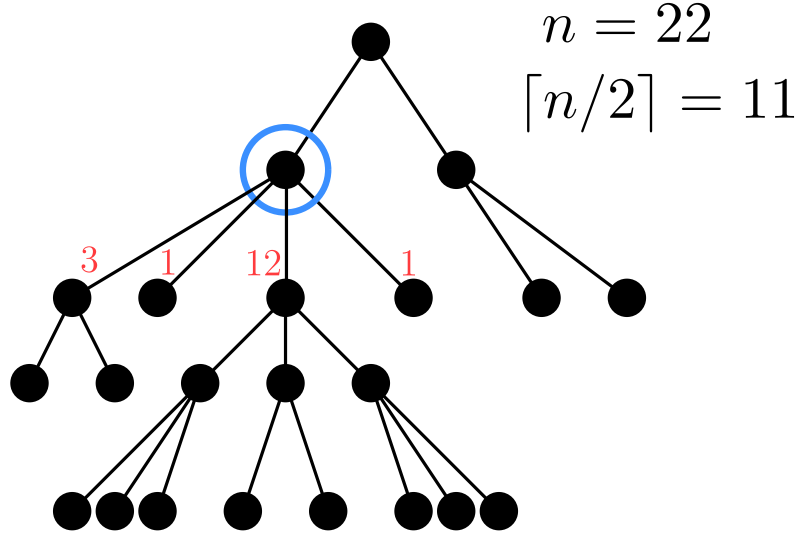

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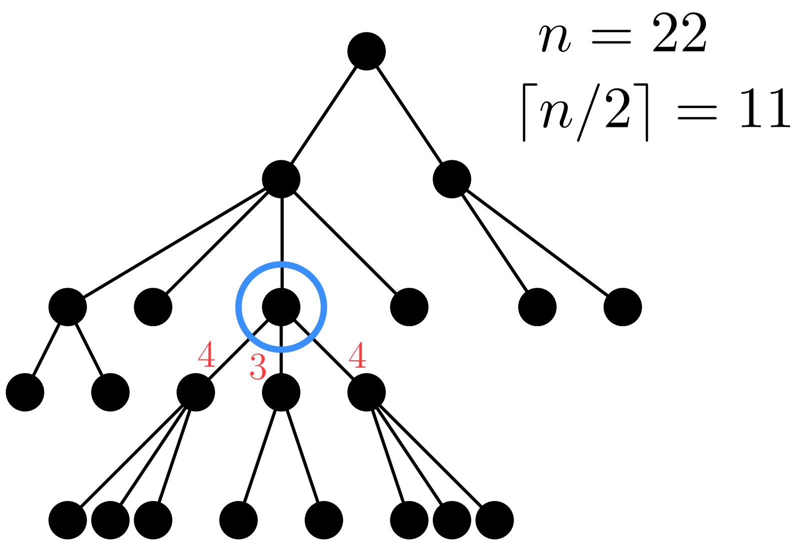

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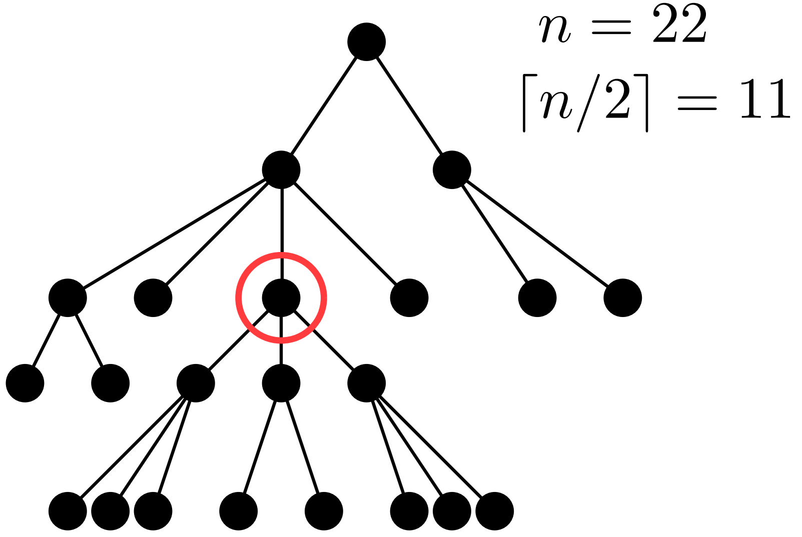

Once we get stuck:

- child subtree sizes $< \lceil n/2 \rceil$
- rooted subtree size $\leq n - \lceil n/2 \rceil \leq \lfloor n/2 \rfloor$

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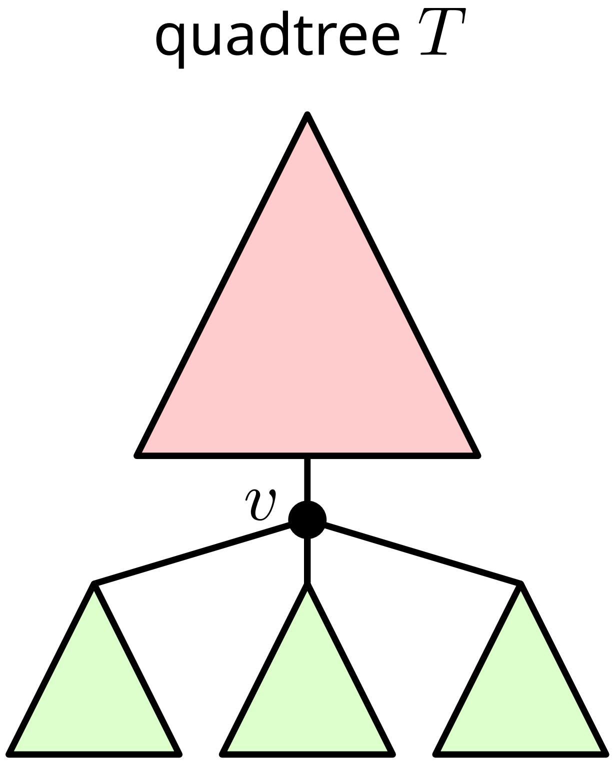

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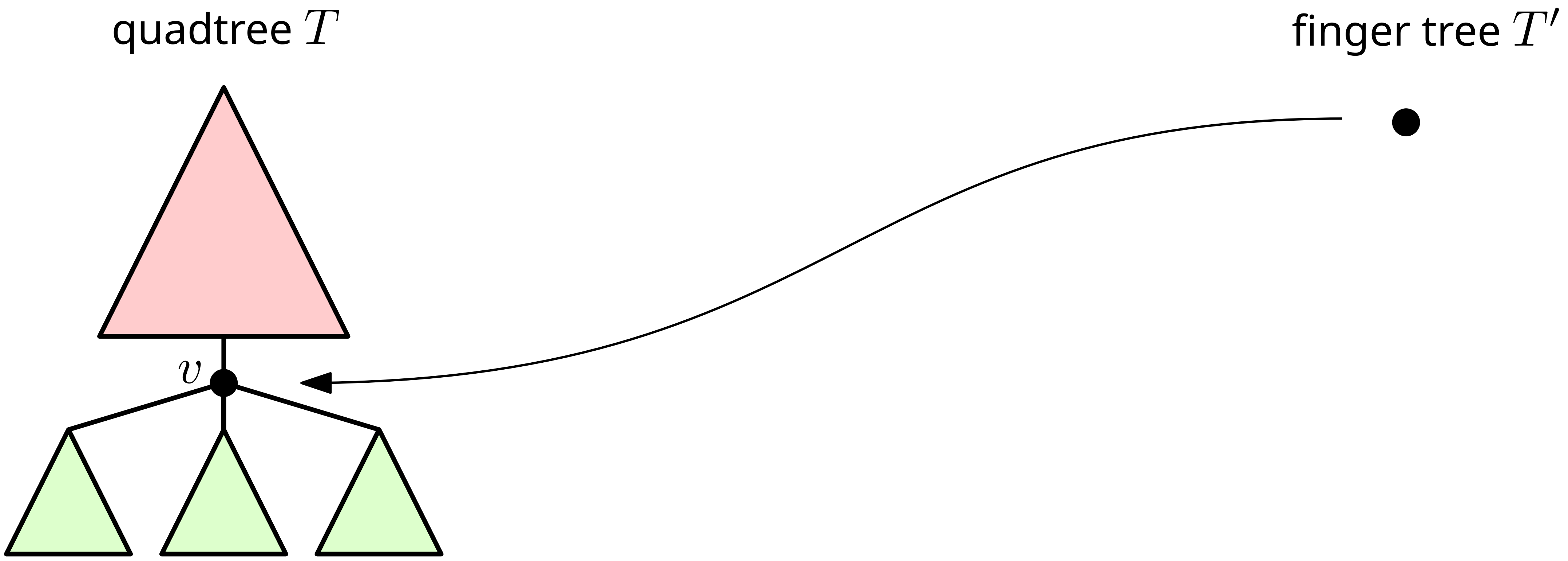

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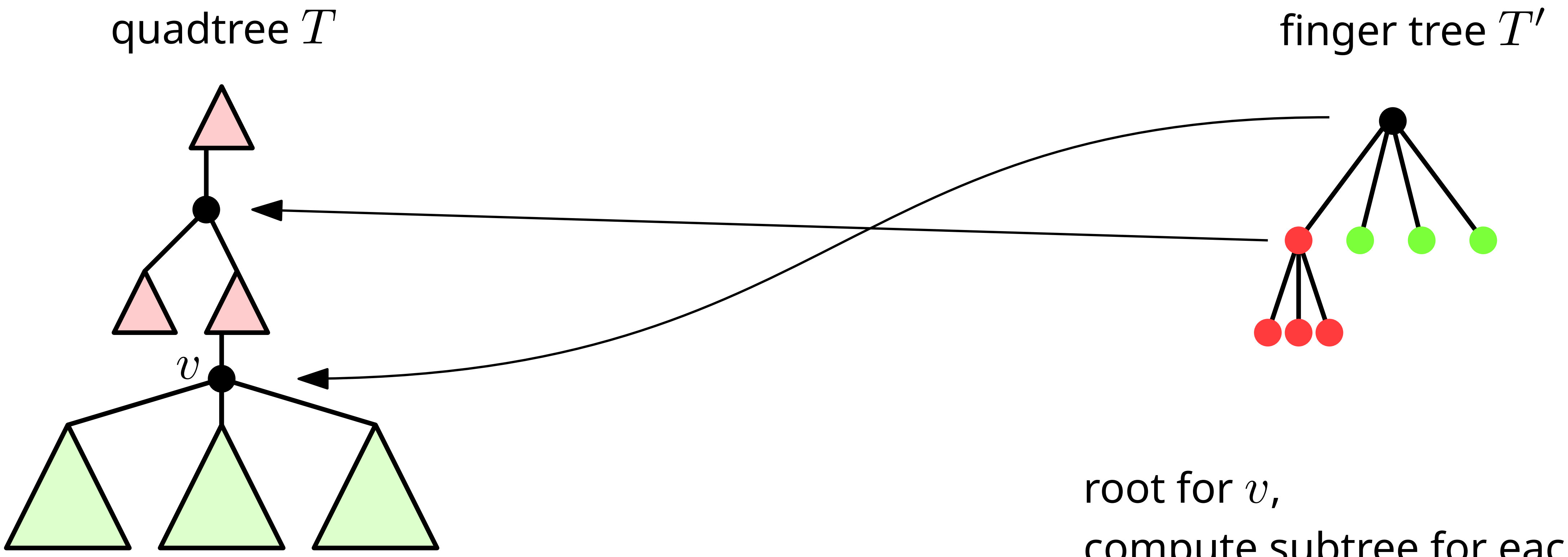

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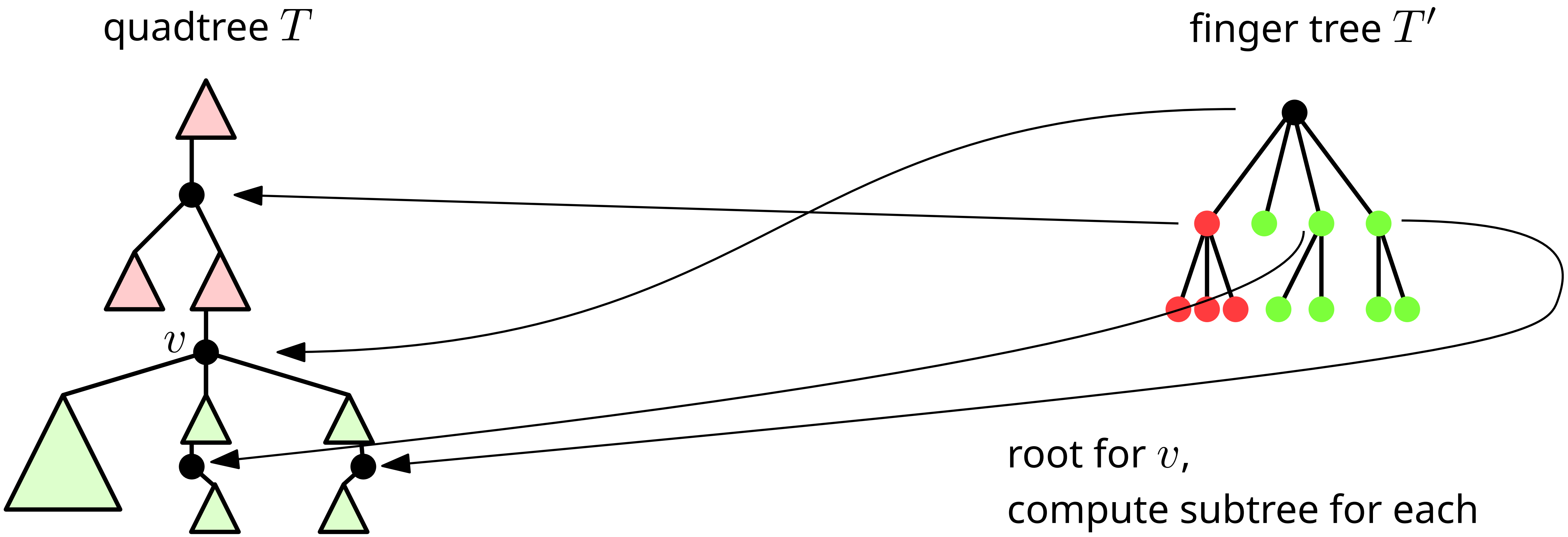

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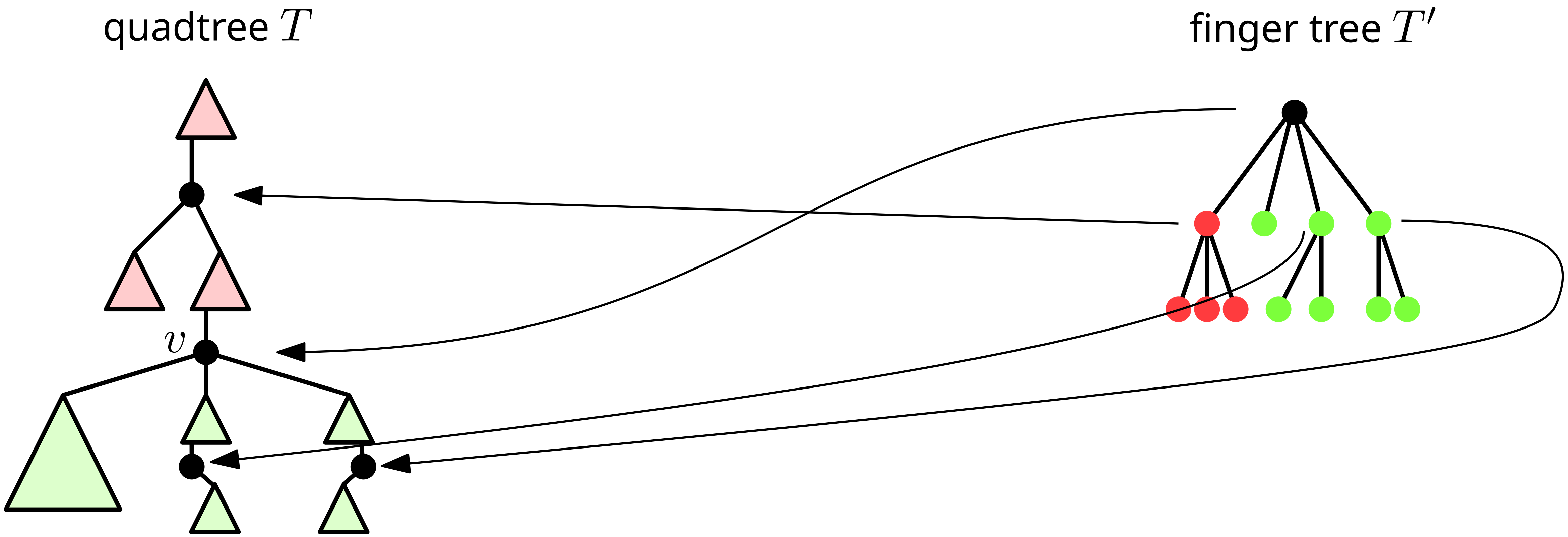


To query for point q, recursively, in time $O($height of $T')$:

- go into red subtree if $q \notin \square_v$
- search all $O(1)$ green subtrees if $q \in \square_v$

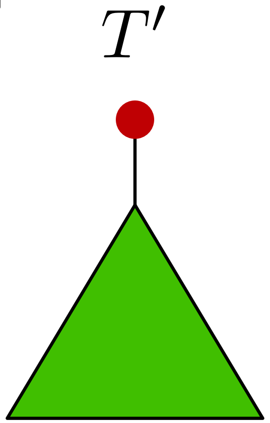
What are the height/query time and the construction time of the finger tree?

Finger trees

Recall that the separator splits T into subtrees of size $\leq \lceil n/2 \rceil$

recurrence for height:

$$\implies H(n) \leq 1 + H(\lceil n/2 \rceil) = O(\log n)$$



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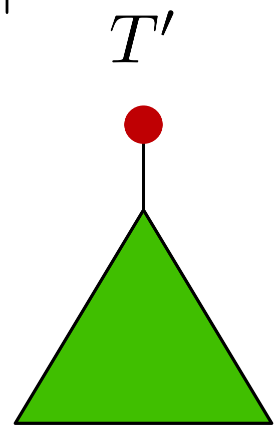

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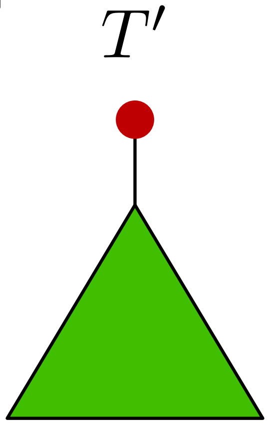
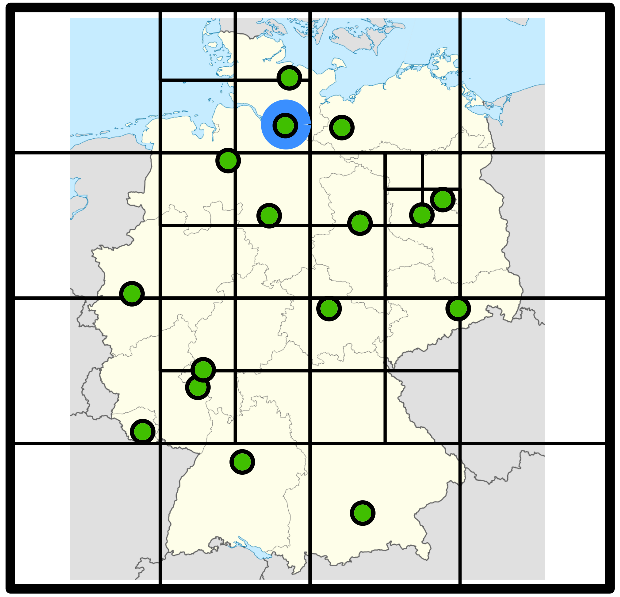


Construction time $T(n) = O(n) + \sum_{i=1}^{t} T(n_i)$ where $n_1 \ldots n_t$ are the sizes of the t subtrees formed after removing the separator.

Since $t = O(1)$ and $n_i \leq \lceil n/2 \rceil$, we have $T(n) = O(n \log n)$

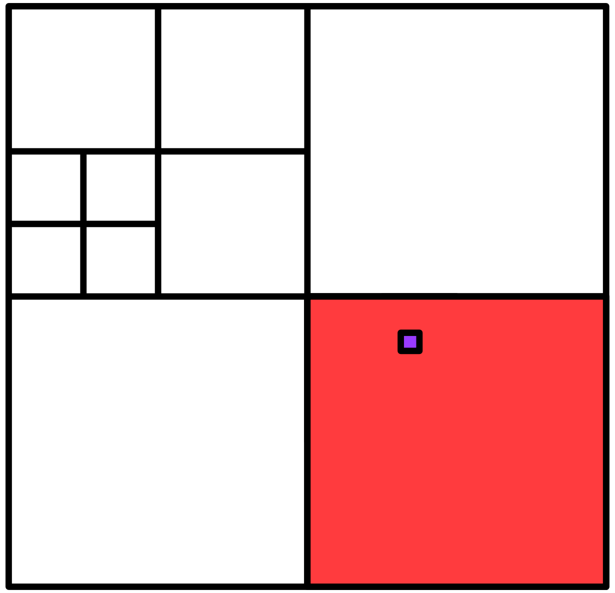
Summary

Normal quadtrees



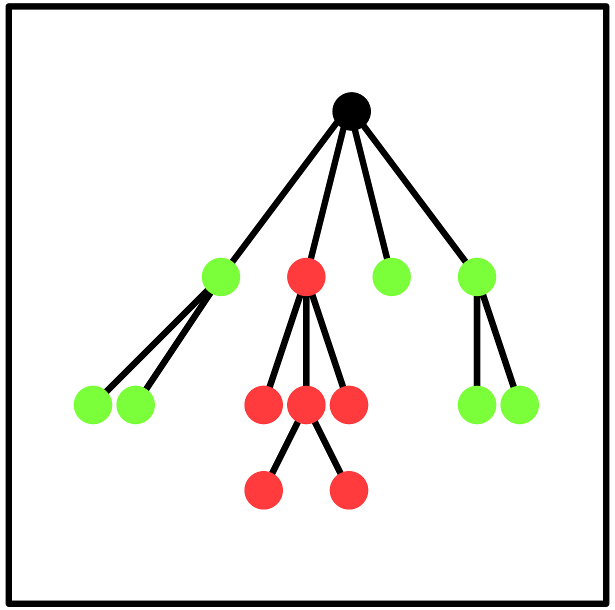
Bounded by spread

Compressed quadtrees



Bounded by number of points

Finger trees



Fast query time

more in book: dynamic quadtrees