Compressed Suffix Trees

Johannes Fischer

What we already have...

- Text + BWT + WT and backwards search

  $\Longrightarrow$ $O(m \lg \sigma)$ counting queries for $P[1,m]$

  - space $O(n \lg \sigma)$ **bits** (text size!)

- + sampled suffix array values

  $\Longrightarrow$ $O(k \lg n)$ for enumerating k occurrences

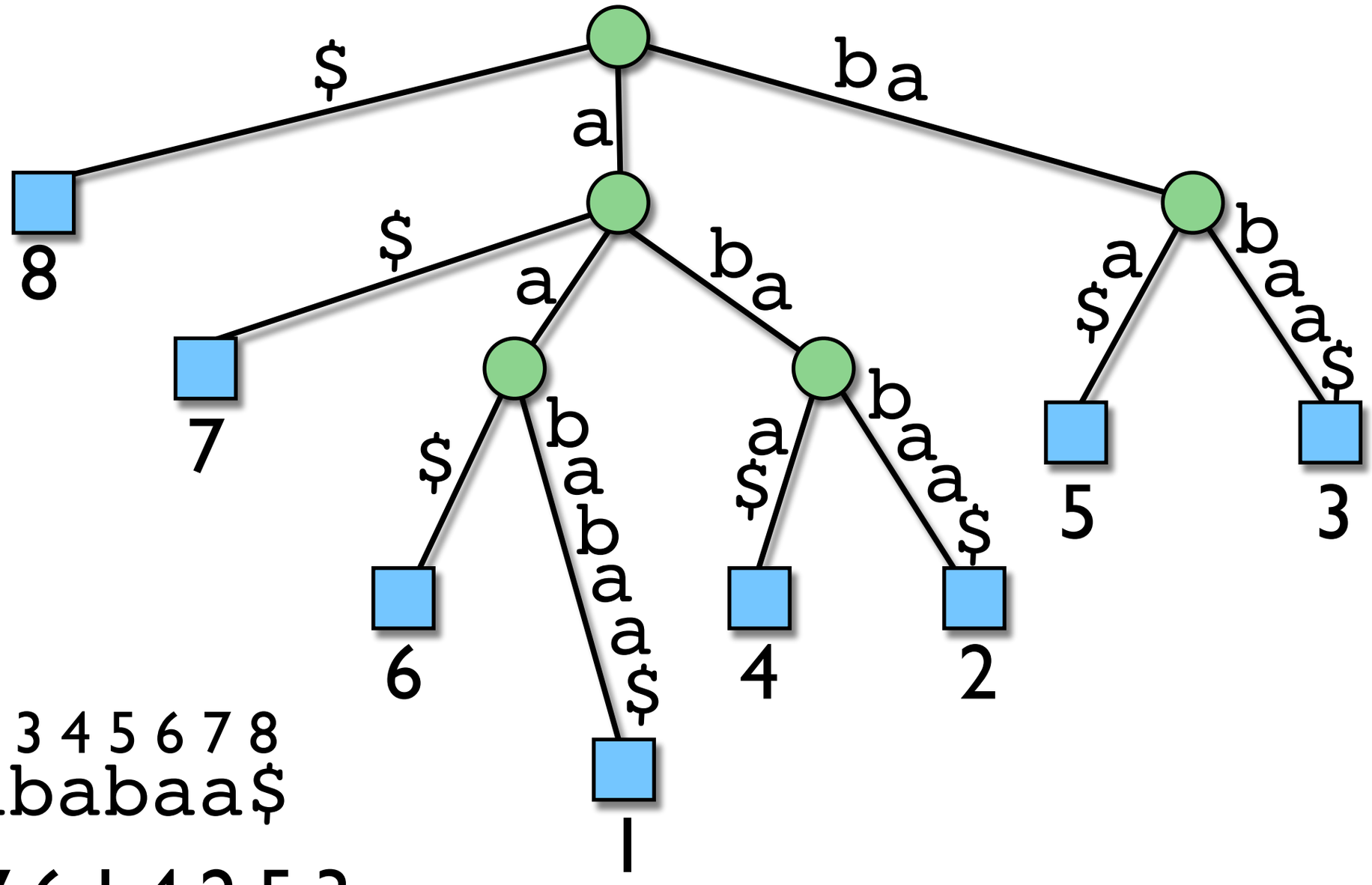
  - can be improved to $O(k \lg^{\varepsilon} n)$ for $\varepsilon<1$

Suffix Tree Functionality

- often, **more functionality** is desired
  - repeat recognition (e.g. $T=\alpha\boldsymbol{\rho}\beta\boldsymbol{\rho}\gamma$)
  - tandem repeats (e.g. $T=\alpha\boldsymbol{\rho}\boldsymbol{\rho}\beta$)
  - longest common substrings
  - matching statistics
  - suffix-prefix matches
  - etc.
- want **suffix tree** functionality!

compress?

Suffix Tree



1 2 3 4 5 6 7 8

T = aababaa$

A = 8,7,6,1,4,2,5,3

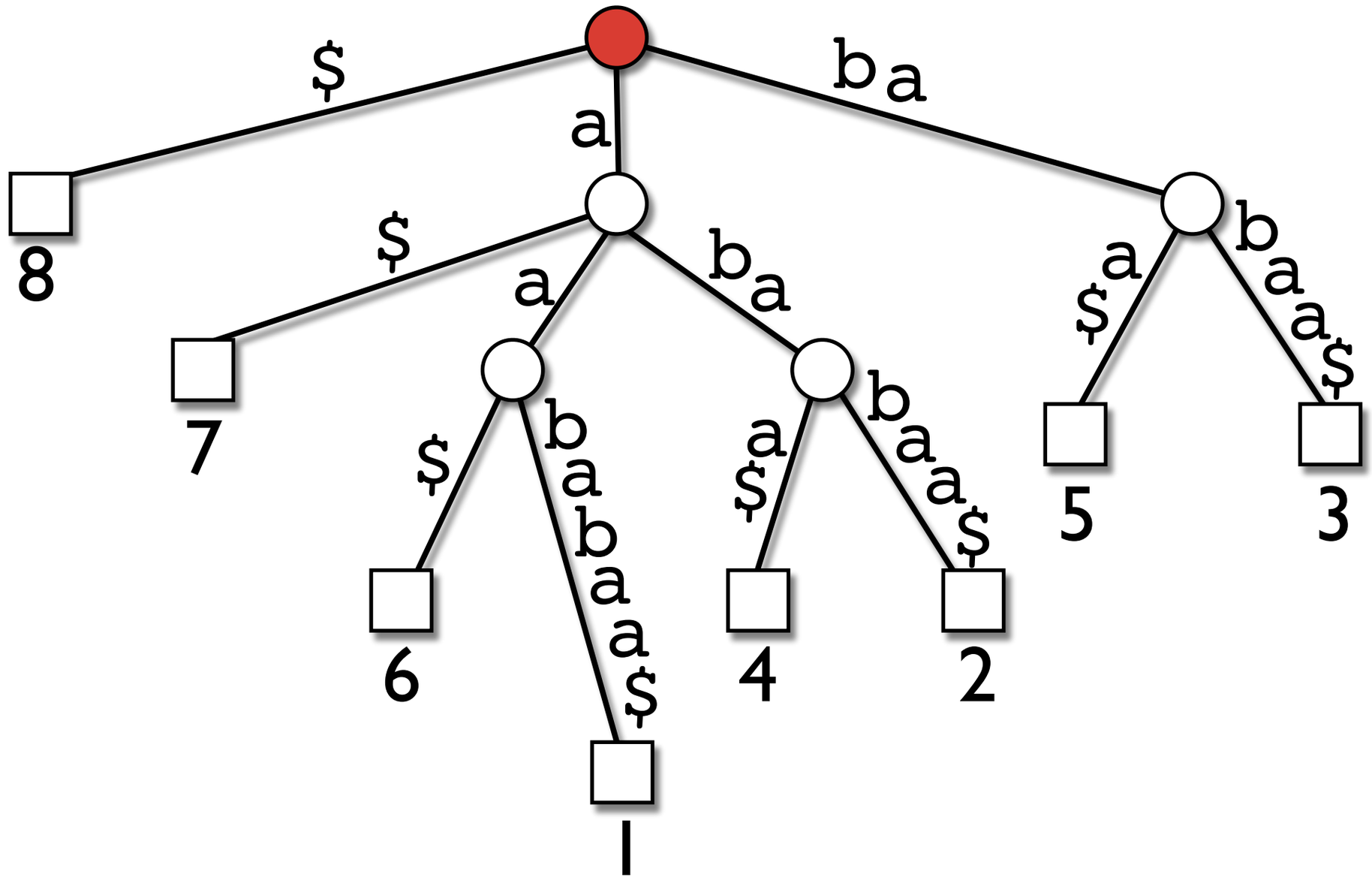
Some real numbers

- **guess**: how much bigger than text is ST?
  A: <10 B: 10-20 C: >20
- Suffix Tree
  - **20-40** times text size !!!
- Text+BWT+WT (incl. rank/select):
  - ≈**3** times text size
- **goal**: drop suffix tree and simulate operations using suffix- and LCP array

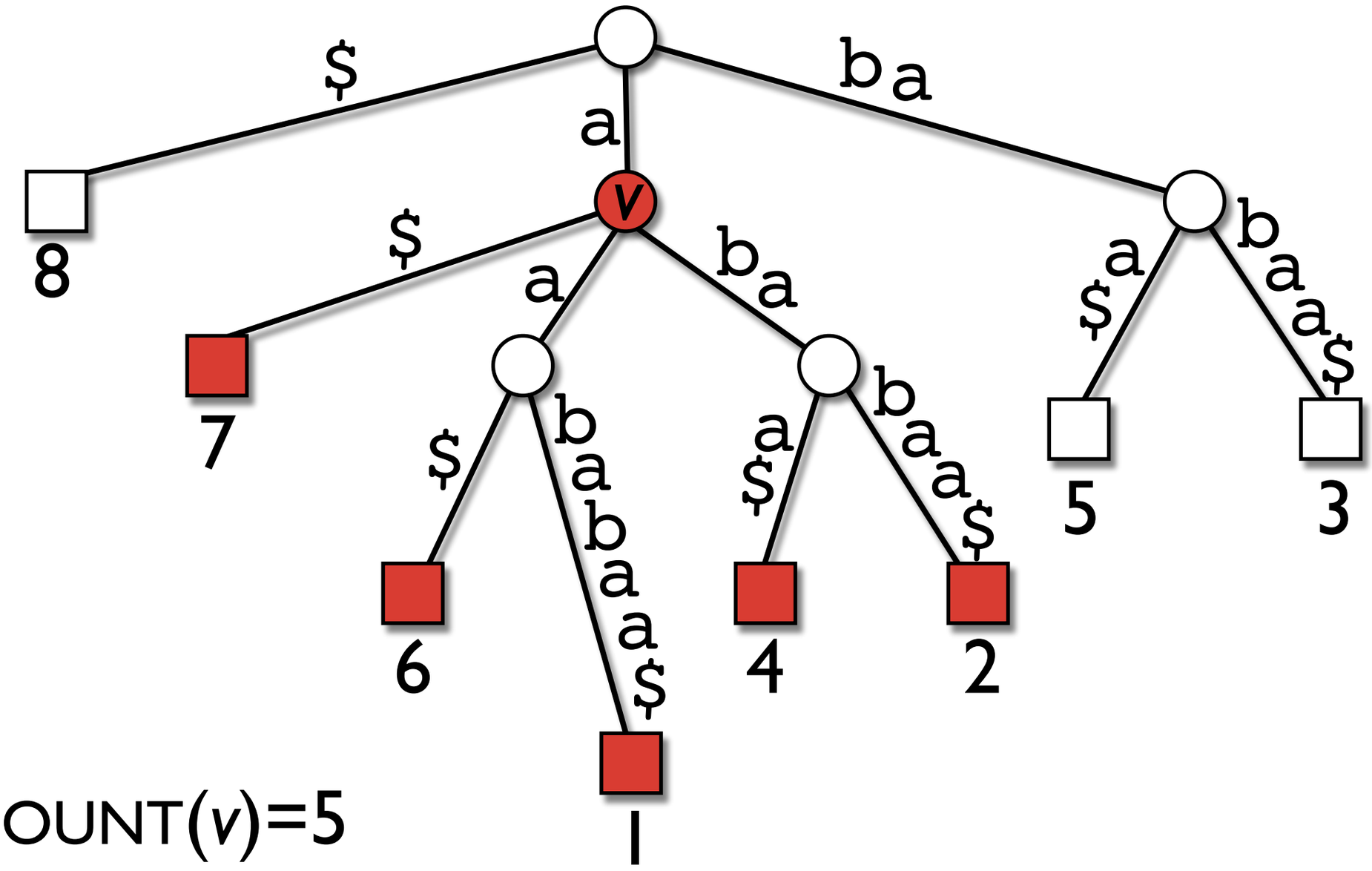
ST Operations

Operation	Description
ROOT()	return root
COUNT(*v*)	count leaves below *v*
ISANCESTOR(*v*,*w*)	true if *v* is an ancestor of *w*
ISLEAF (*v*)	true if *v* is a leaf
LEAFLABEL(*v*)	suffix number represented by leaf *v*
SDEPTH(*v*)	string depth of *v*
PARENT(*v*)	parent node of *v*
FIRSTCHILD(*v*)	first (alphabetically smallest) child of *v*
NEXTSIBLING(*v*)	next sibling of *v*
EDGELABEL(*v*,*i*)	*i*'th letter on the edge leading to *v*
LCA(*v*,*w*)	lowest common ancestor of *v* and *w*

ROOT()

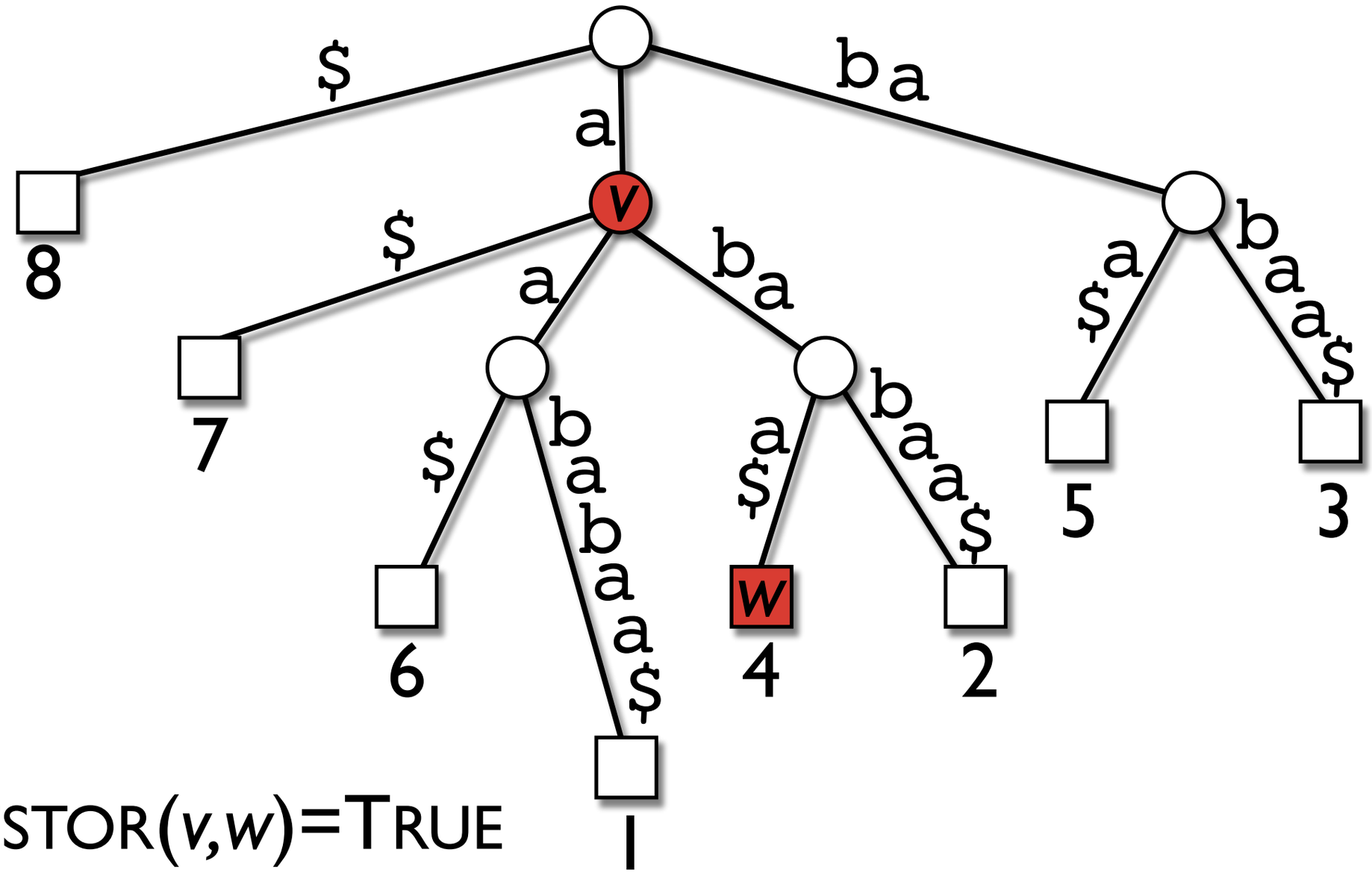


COUNT(*v*)



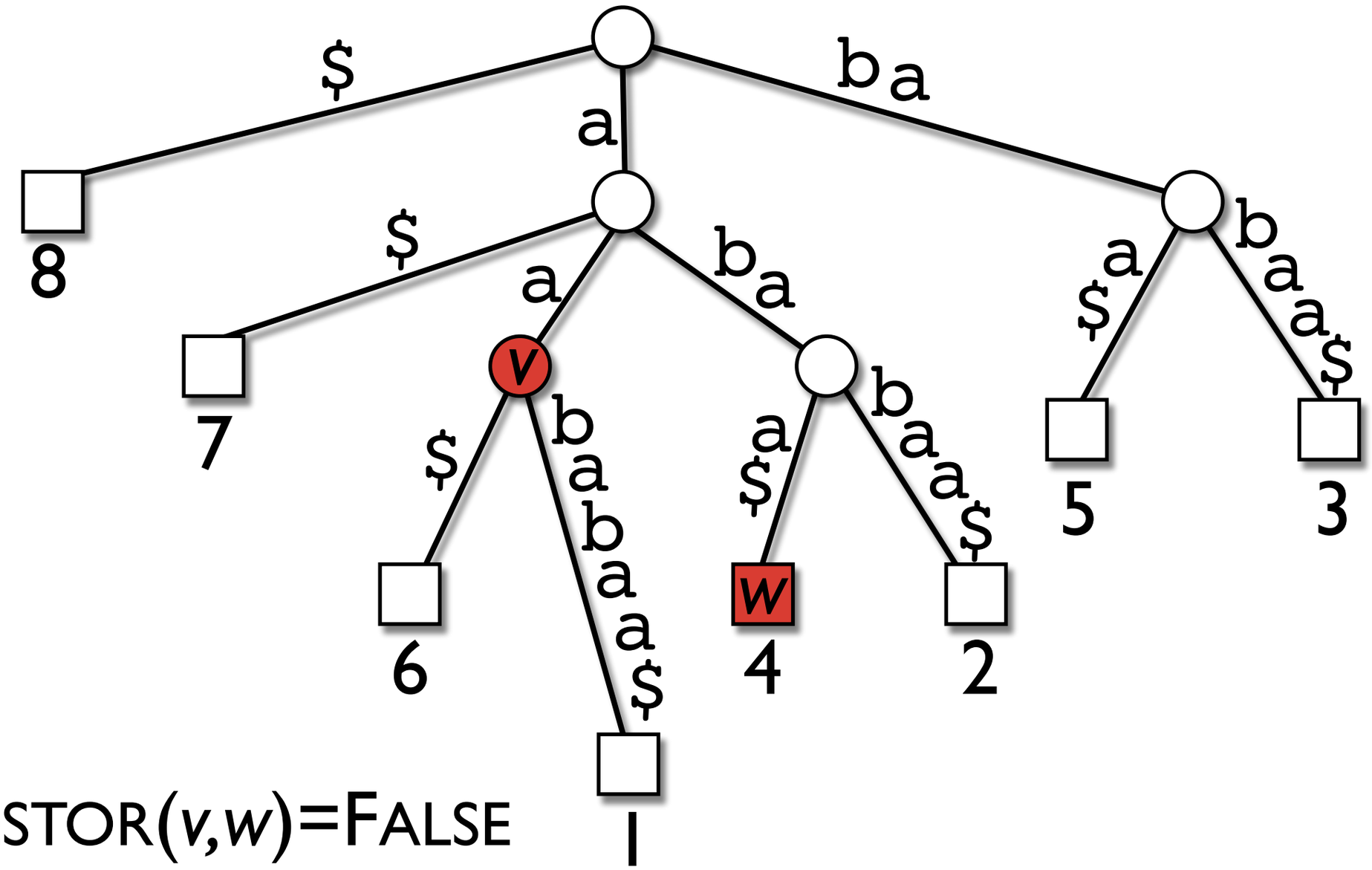
COUNT(*v*)=5

IsAncestor(*v*,*w*)



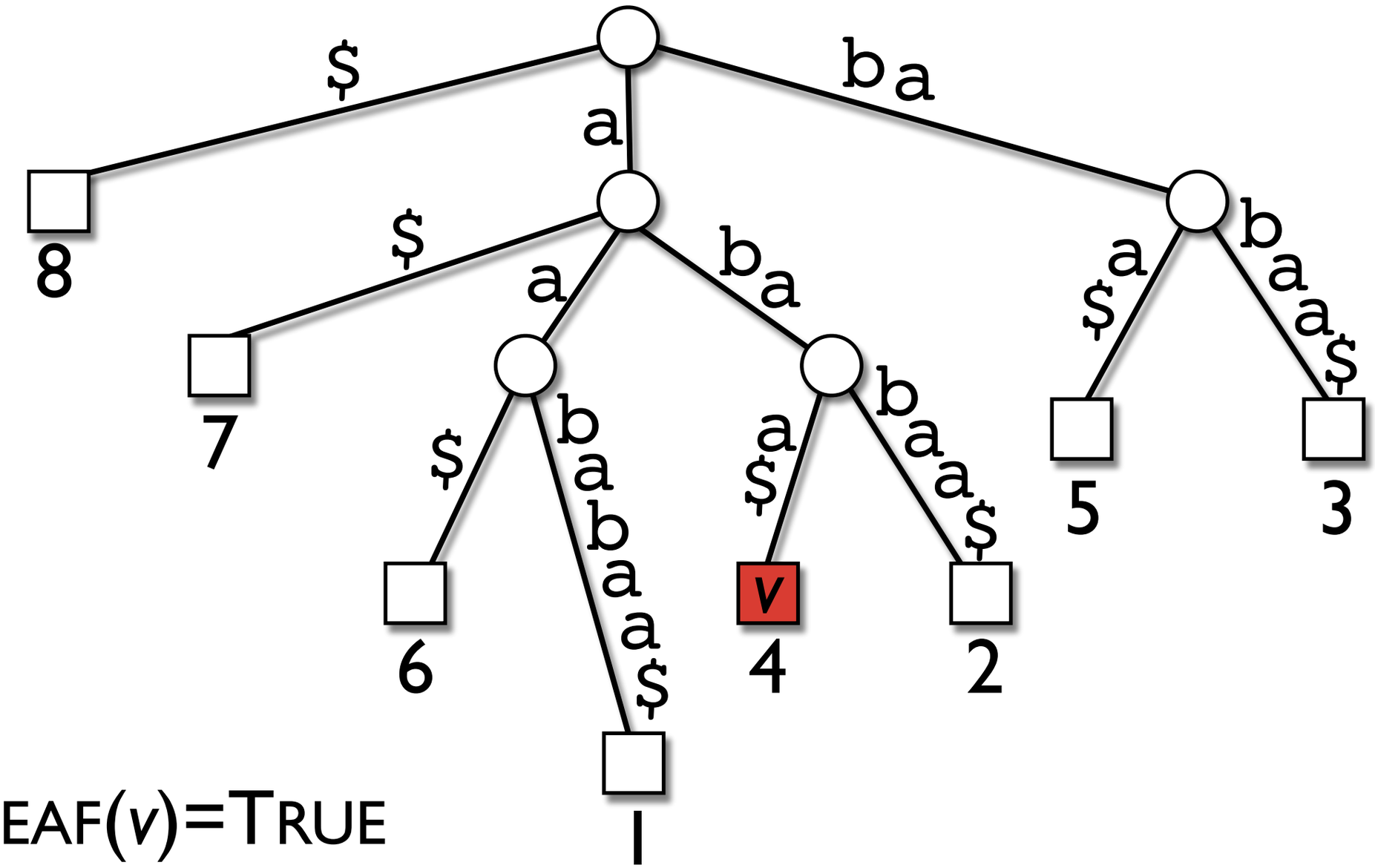
IsAncestor(*v*,*w*)=True

IsAncestor(*v*,*w*)



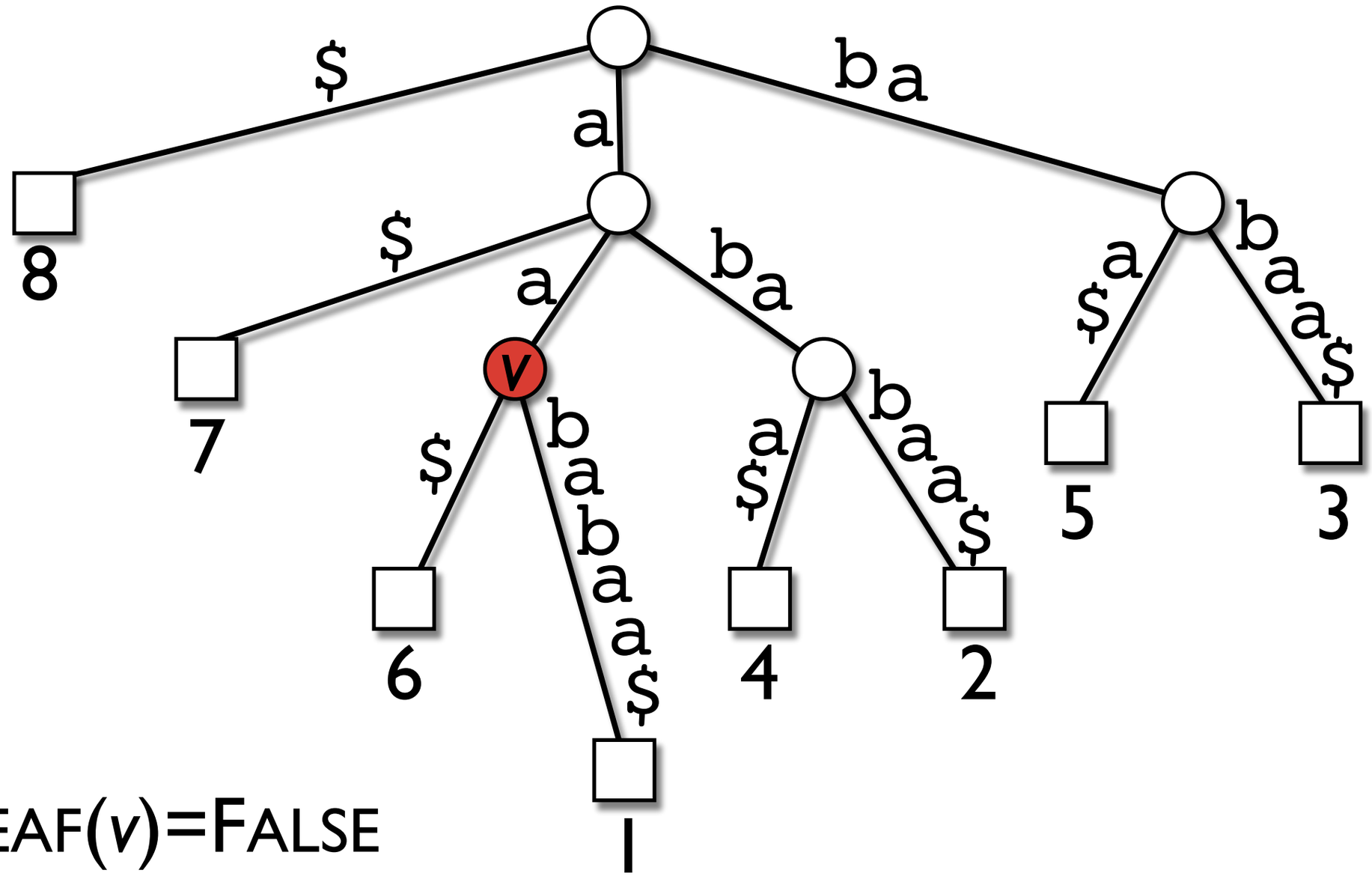
IsAncestor(*v*,*w*)=False

IsLeaf(*v*)



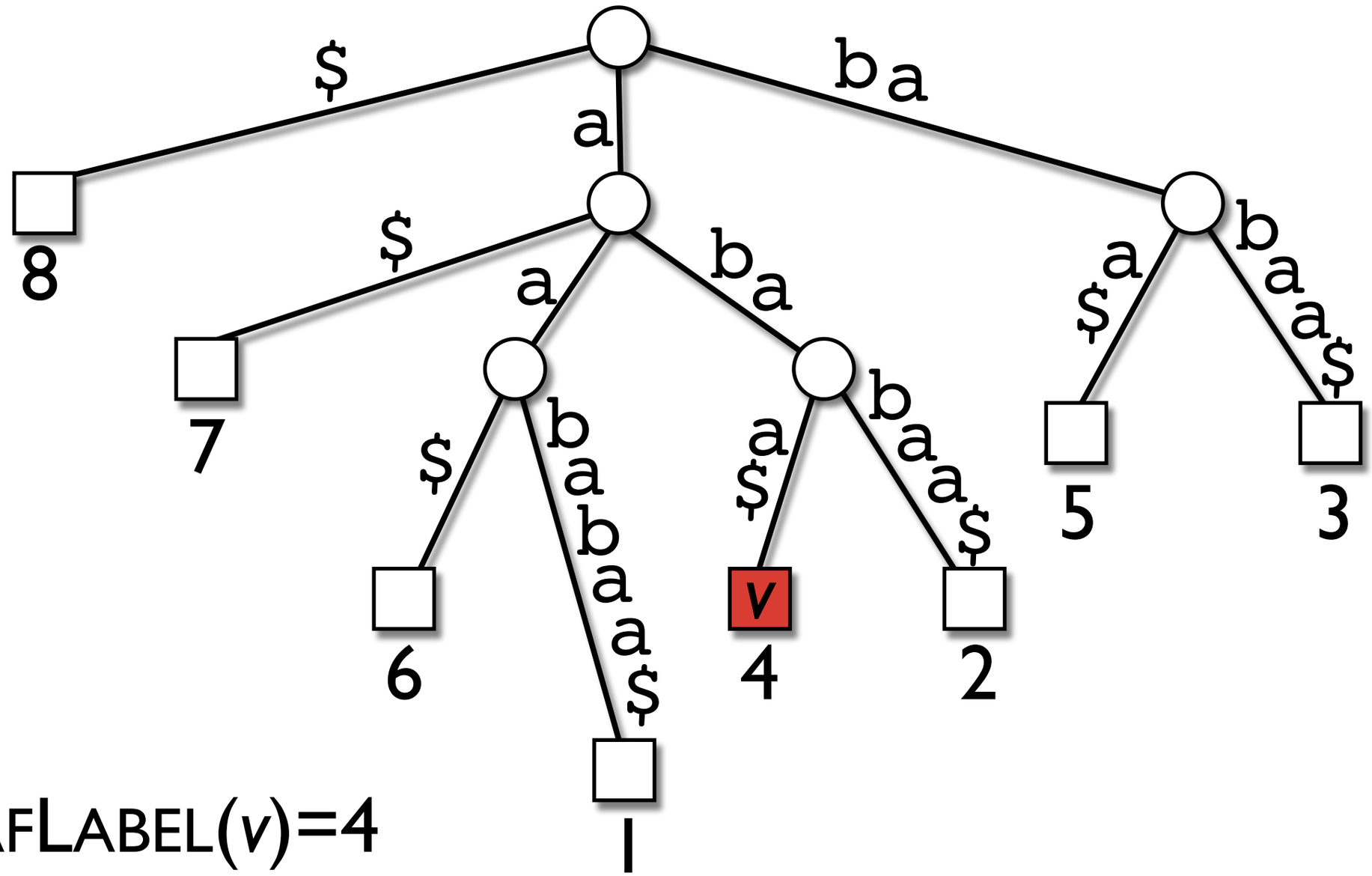
IsLeaf(*v*)=True

IsLeaf(*v*)



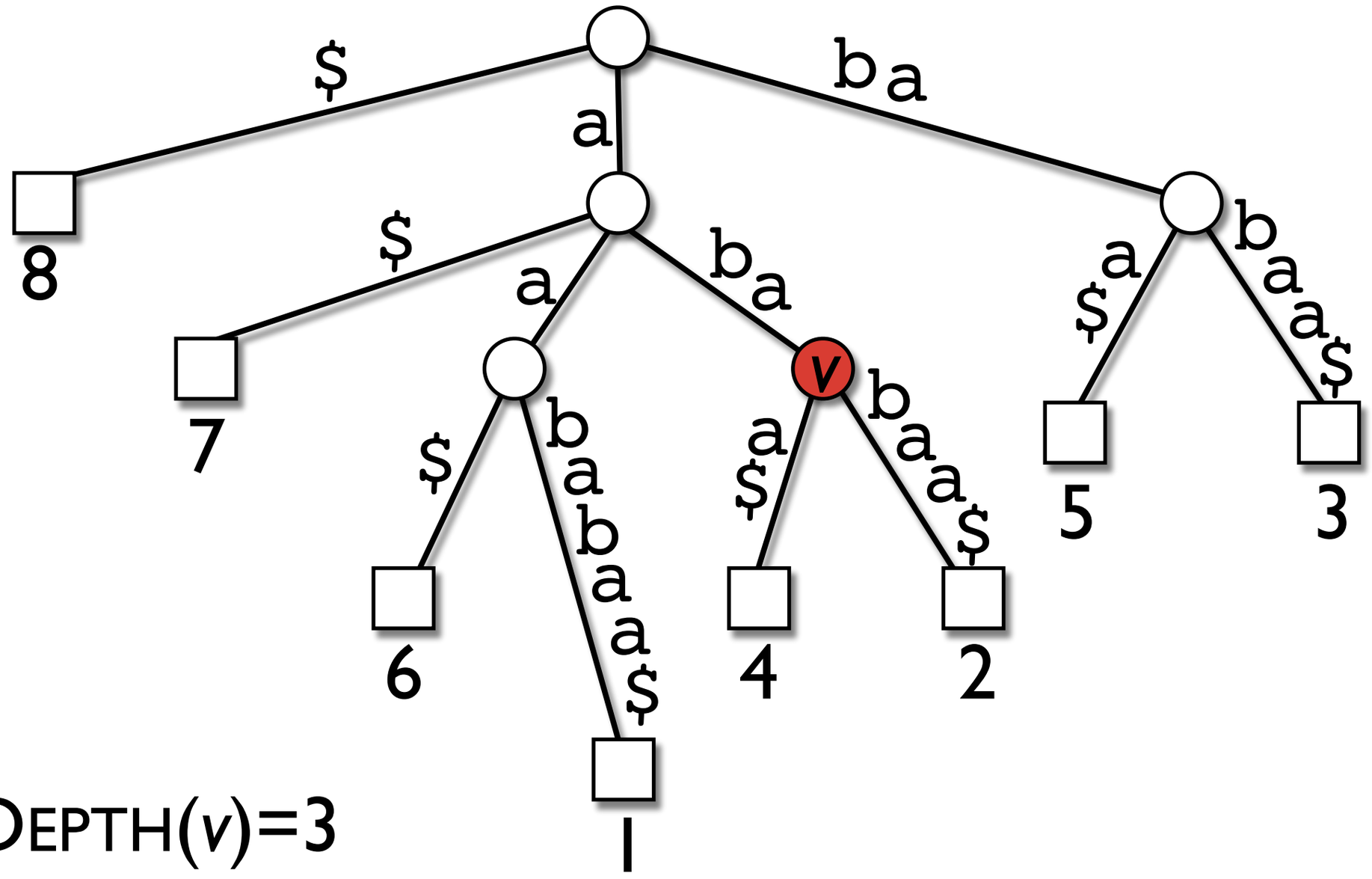
IsLeaf(*v*)=False

LEAFLABEL(*v*)



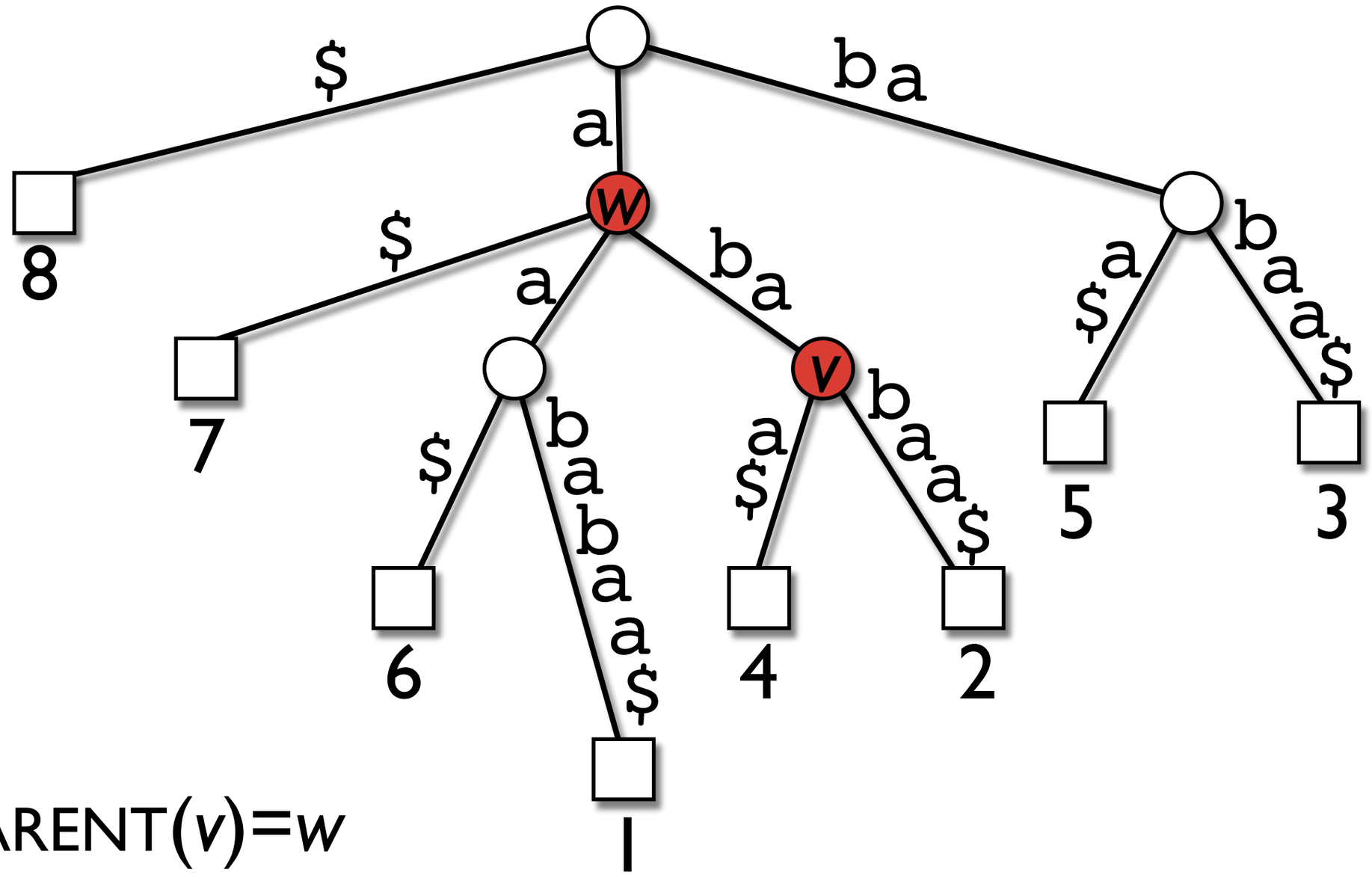
LEAFLABEL(*v*)=4

SDEPTH(*v*)



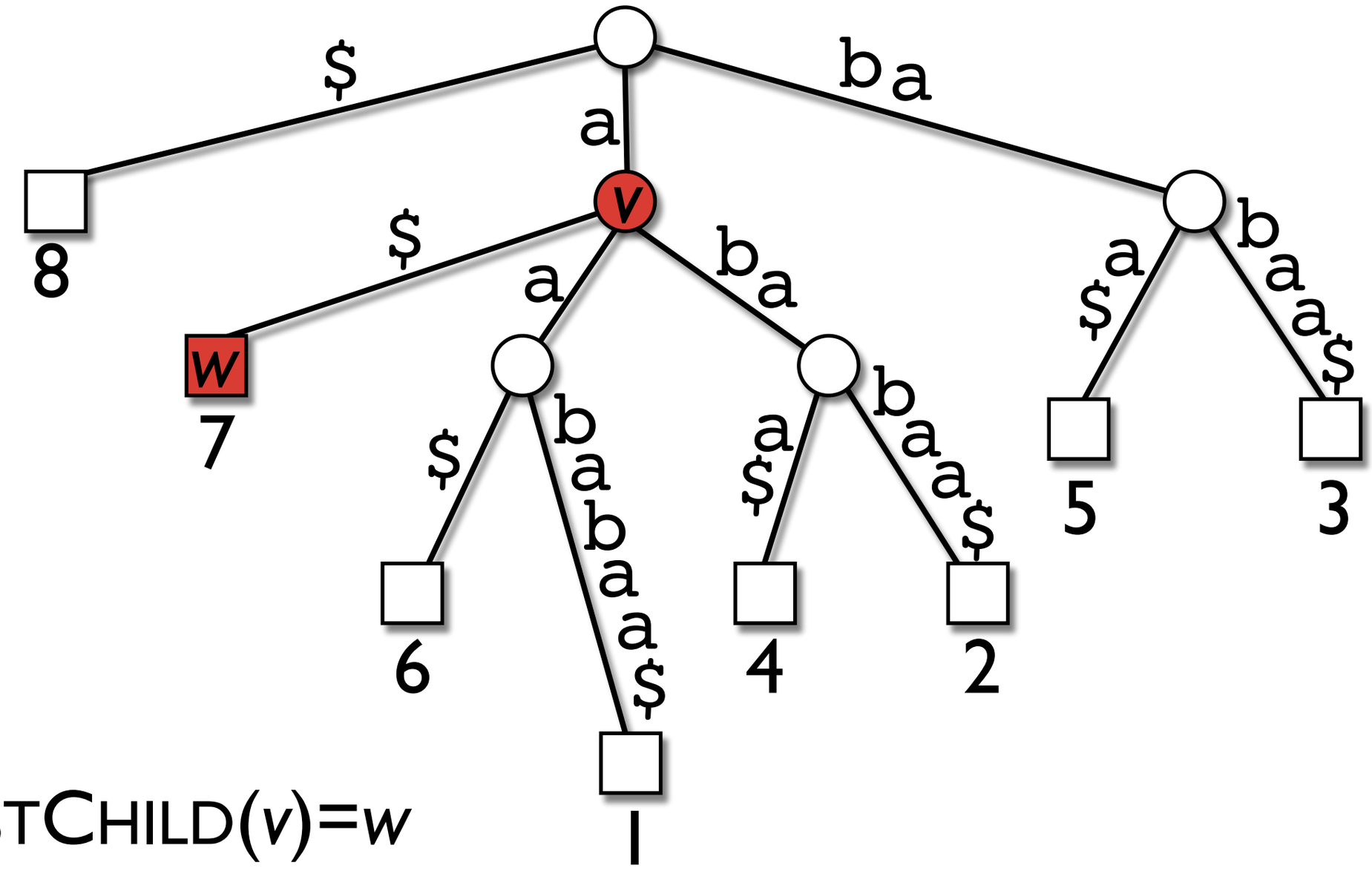
SDEPTH(*v*)=3

PARENT(v)



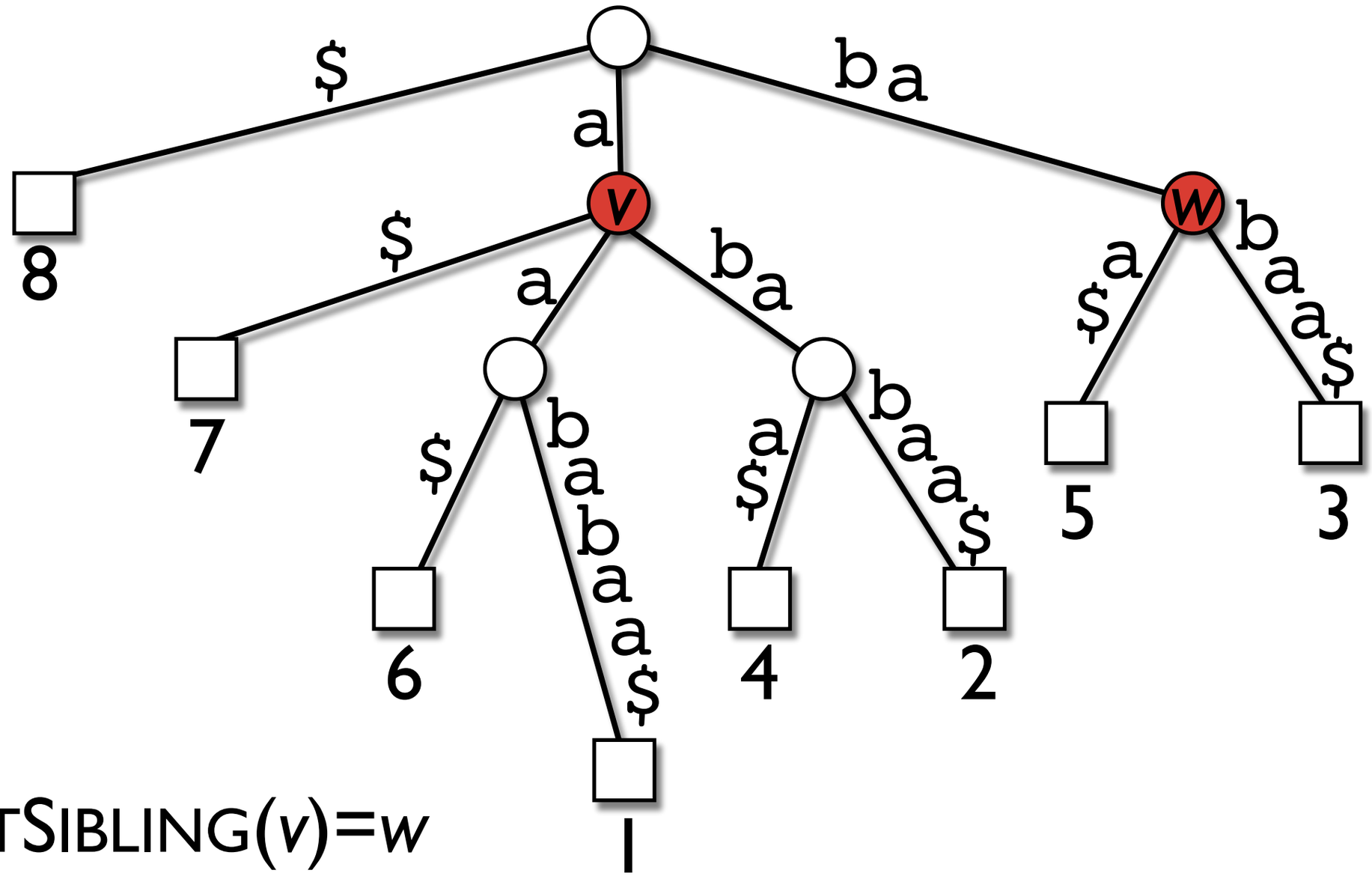
PARENT(v)=w

FIRSTCHILD(*v*)



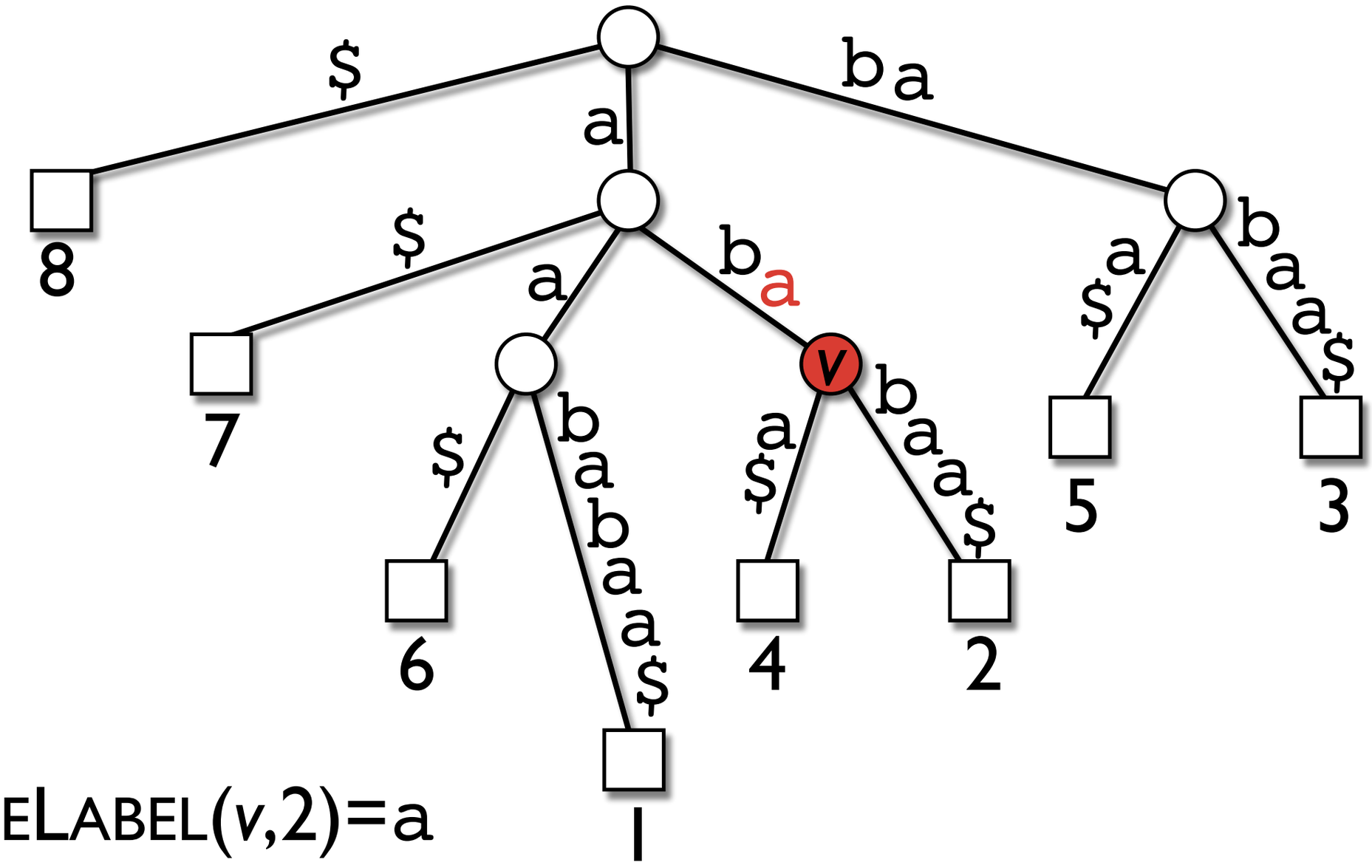
FIRSTCHILD(*v*)=*w*

NEXTSIBLING(*v*)



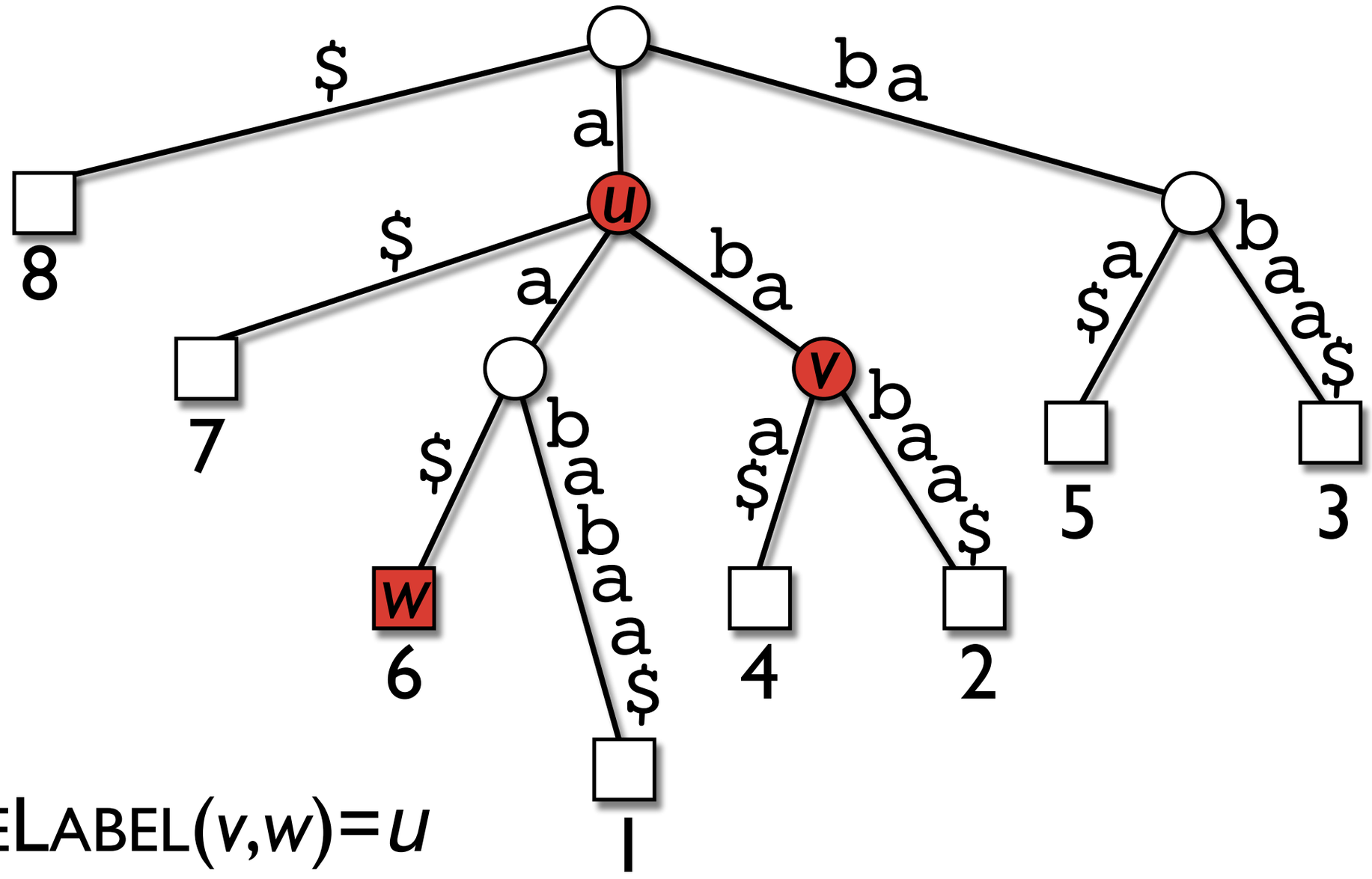
NEXTSIBLING(*v*)=*w*

EDGELABEL(*v*,*i*)



EDGELABEL(*v*,2)=a

LCA(*v*,*w*)



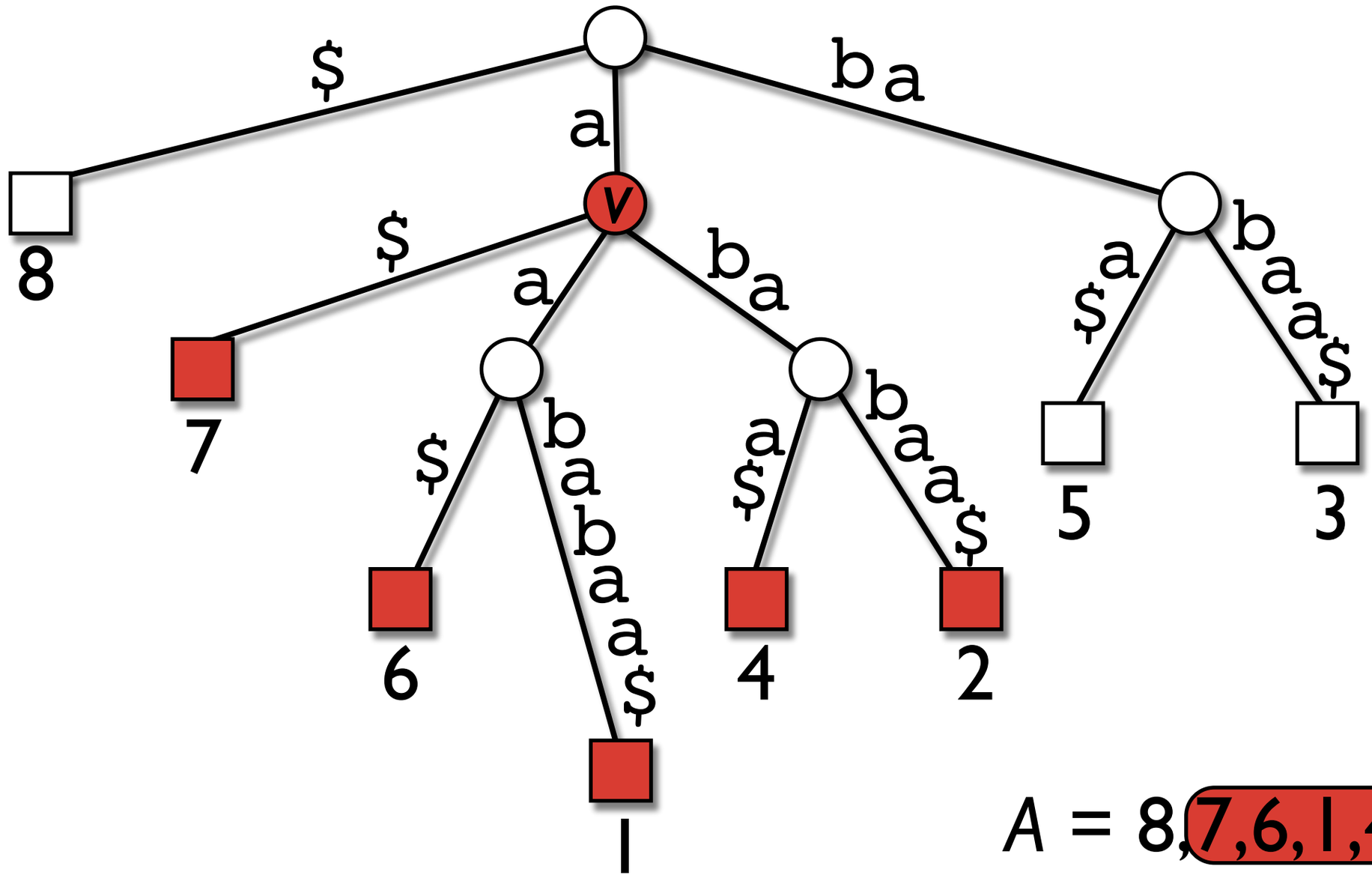
EDGELABEL(*v*,*w*)=*u*

Goal

drop suffix tree and **simulate** operations

using suffix- and LCP array

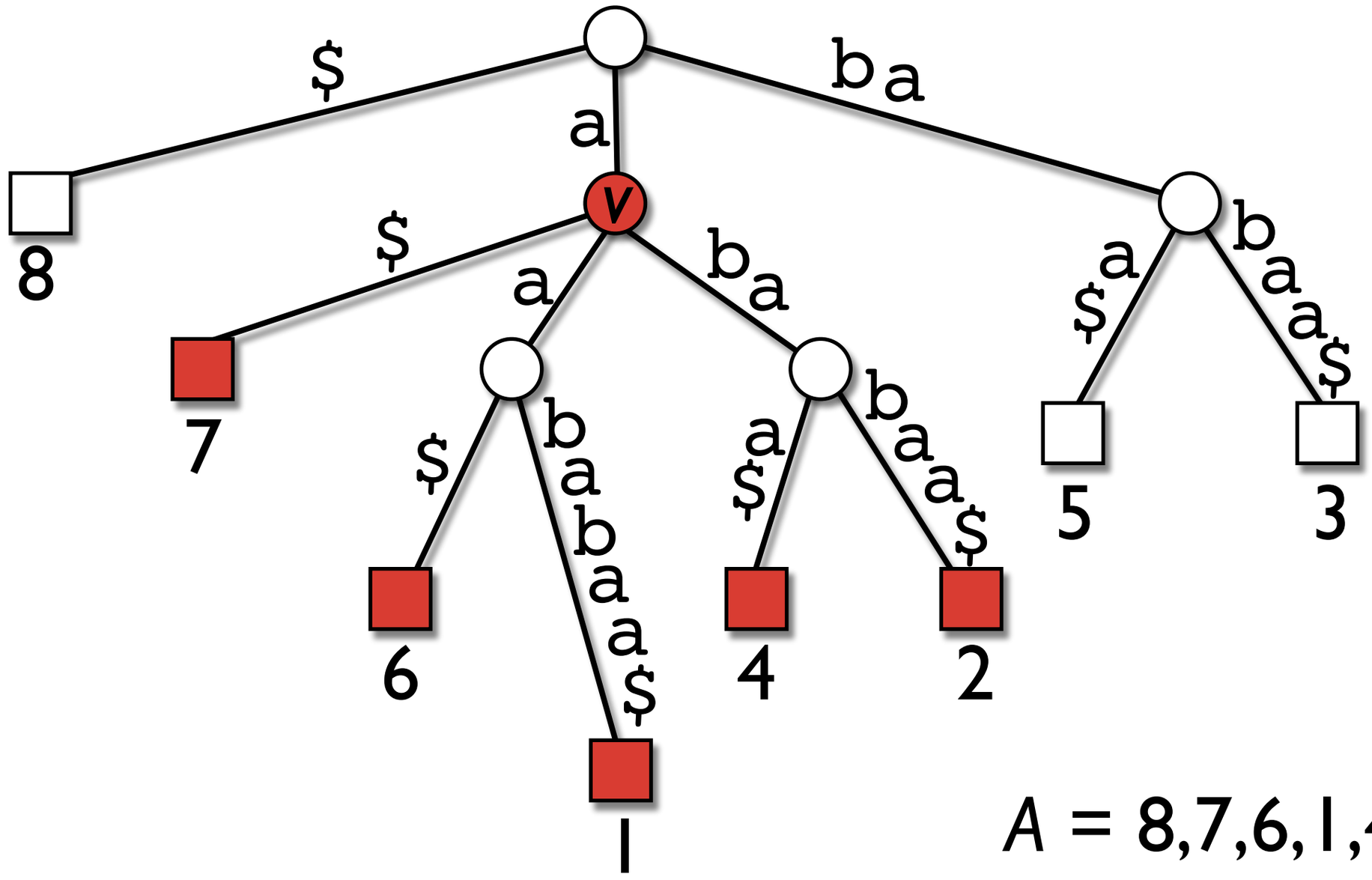
Represent Nodes by Intervals



$A = 8,7,6,1,4,2,5,3$

$H = -1,0,1,2,1,3,0,2$

Represent Nodes by Intervals

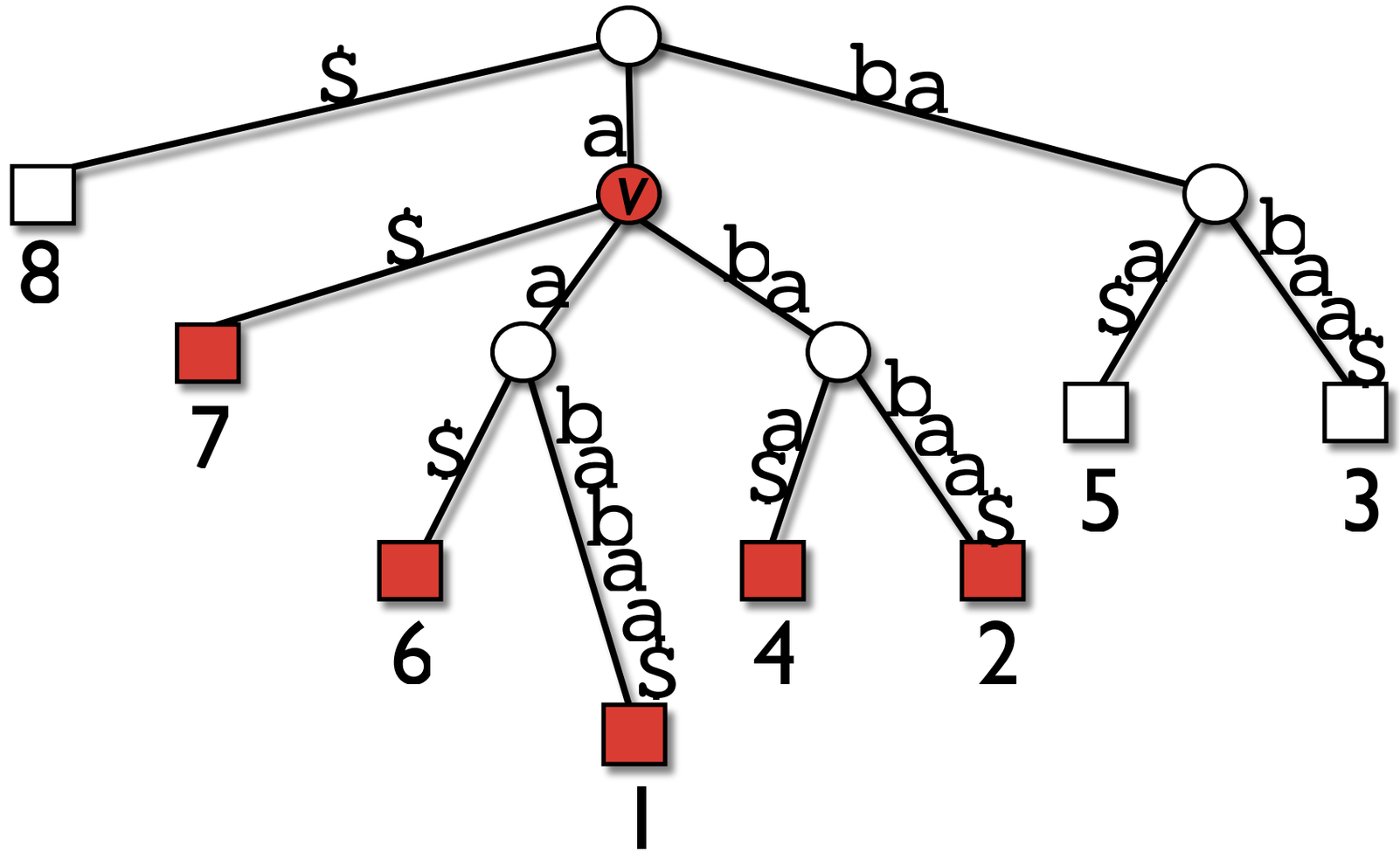


$A = 8,7,6,1,4,2,5,3$

$H = -1,0,1,2,1,3,0,2$

Intervals $[v_l, v_r]$ in H

1. $H[i] \geq \text{SDEPTH}(v)$
   $\forall\ v_l < i \leq v_r$



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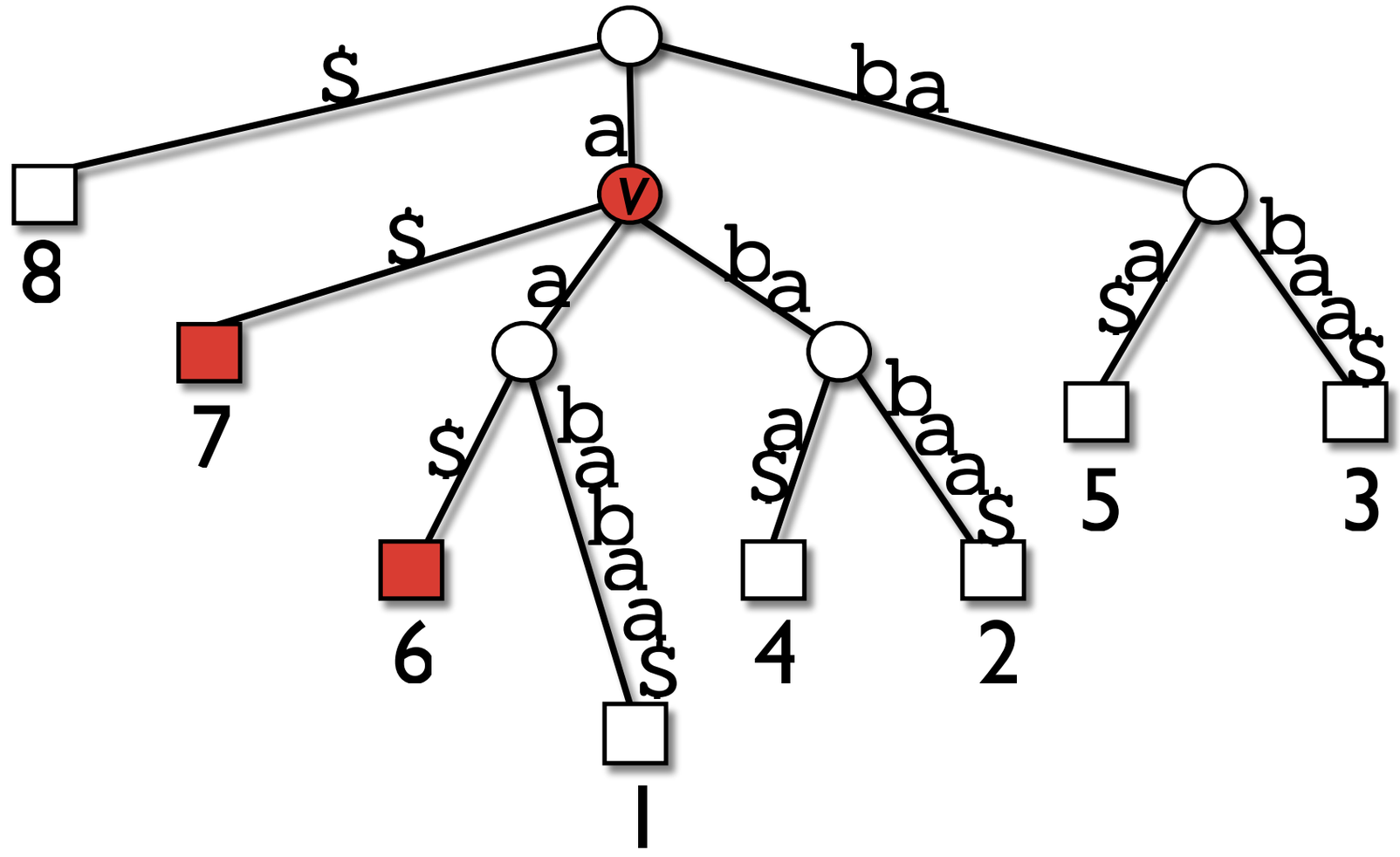

$A = 8,7,6,1,4,2,5,3$
(v_l … v_r)

$H = -1,0,1,2,1,3,0,2$

Intervals $[v_l, v_r]$ in H

1. $H[i] \geq \text{SDEPTH}(v)$ 8
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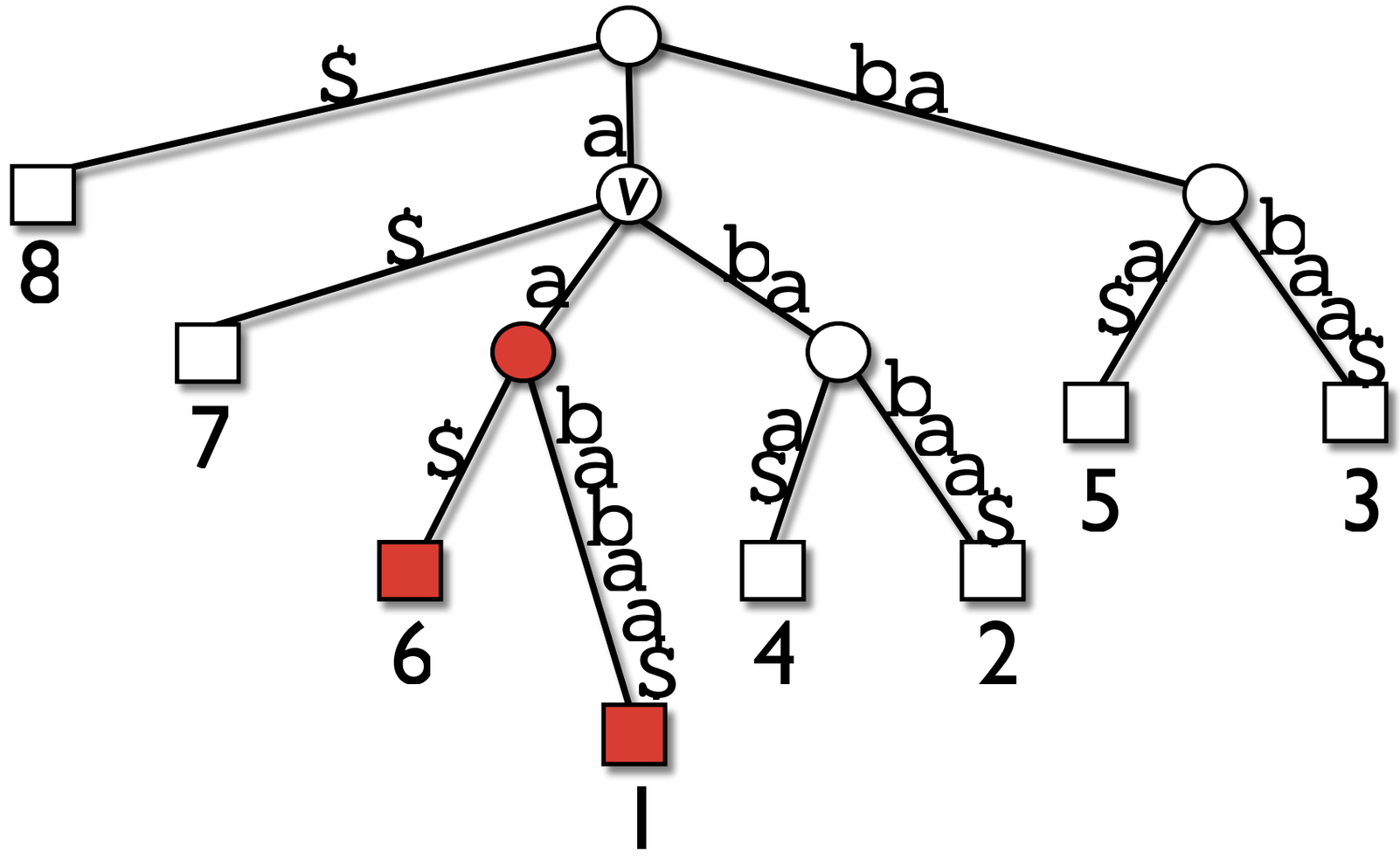

$A = 8,7,6,1,4,2,5,3$
v_l v_r

$H = -1,0,1,2,1,3,0,2$

Intervals $[v_l, v_r]$ in H

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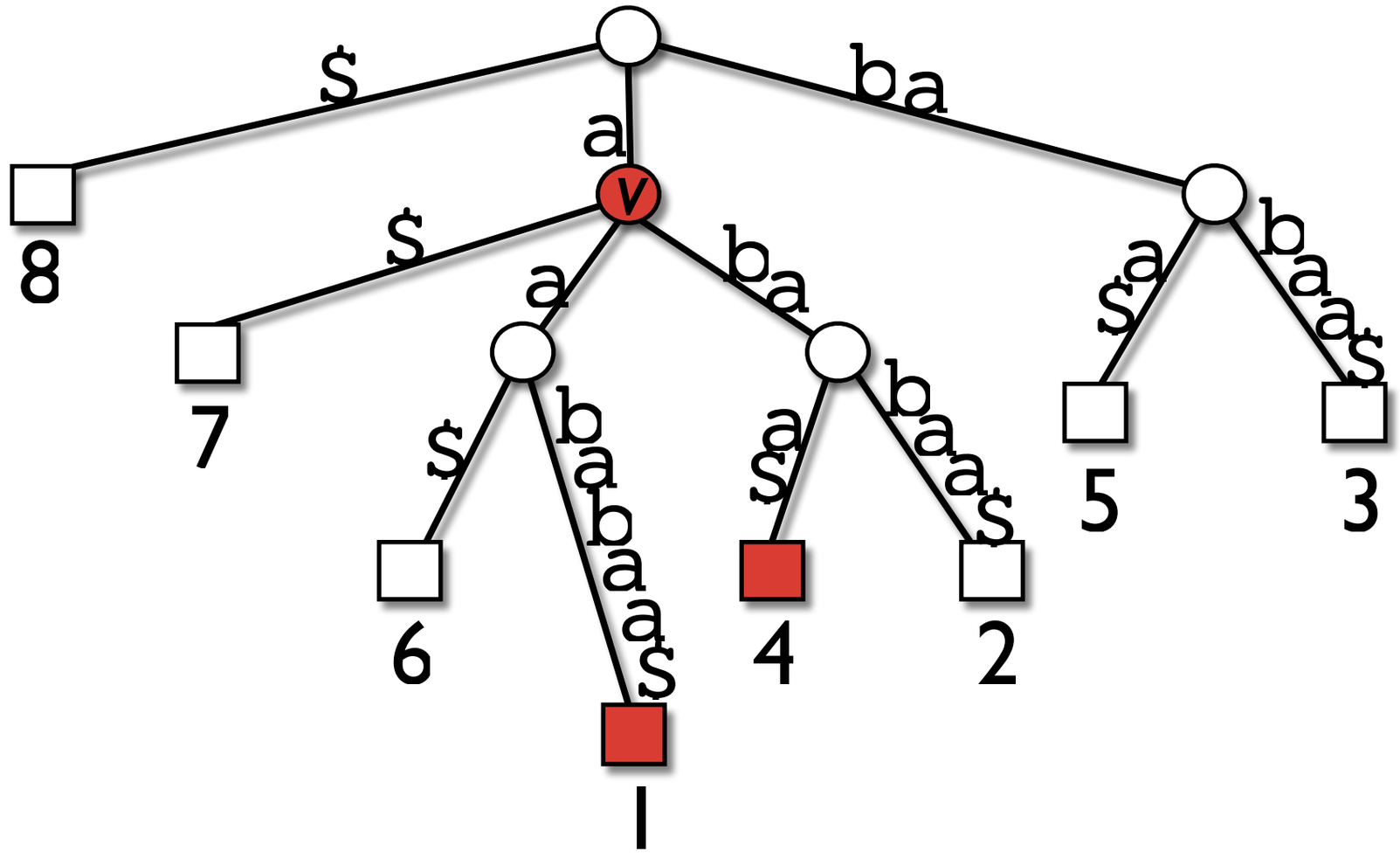
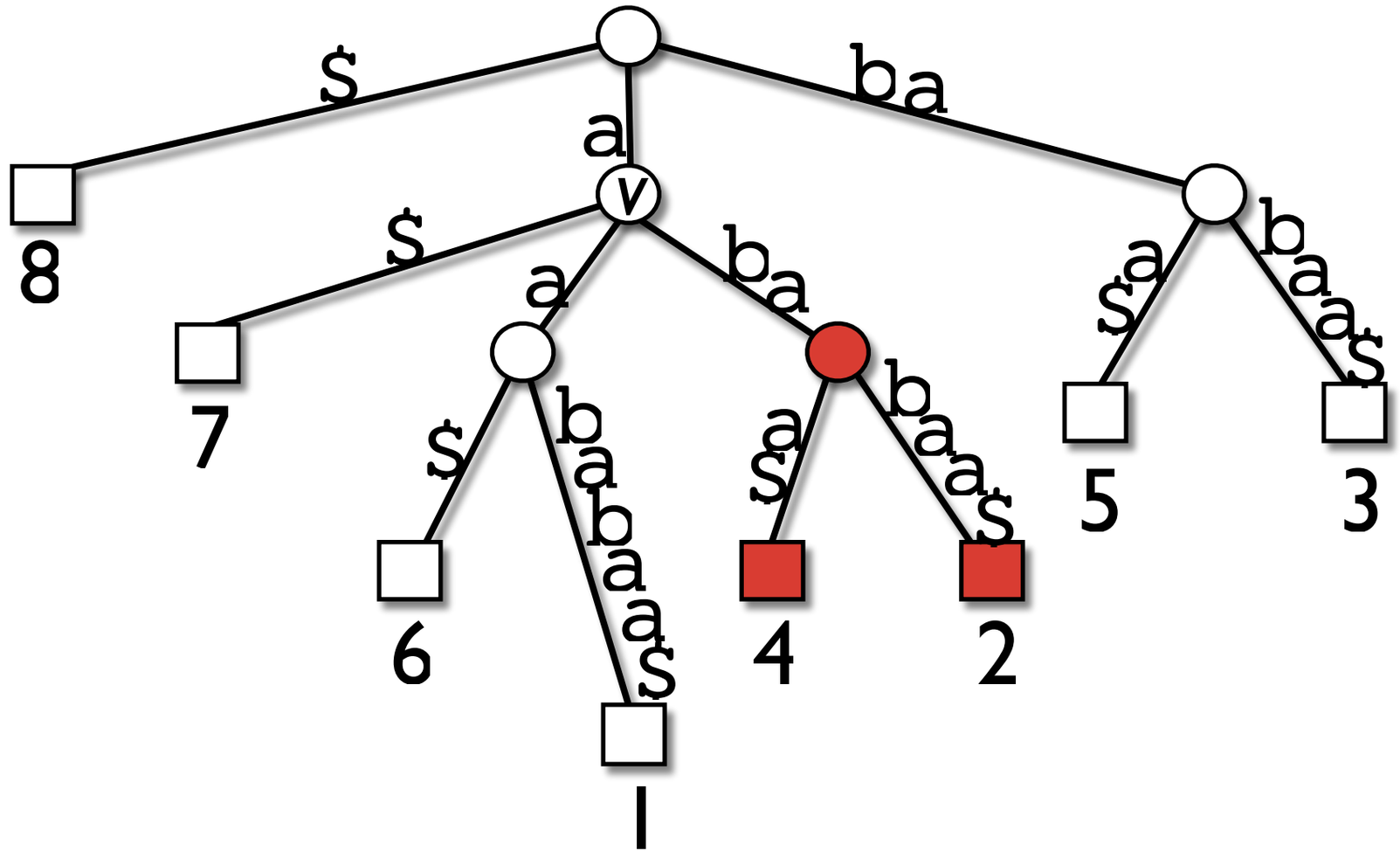


$A = 8,7,6,1,4,2,5,3$
(v_l = 7, v_r = 2)

$H = -1,0,1,2,1,3,0,2$

Intervals $[v_l, v_r]$ in H

1. $H[i] \geq \text{SDEPTH}(v)$
   $\forall\ v_l < i \leq v_r$



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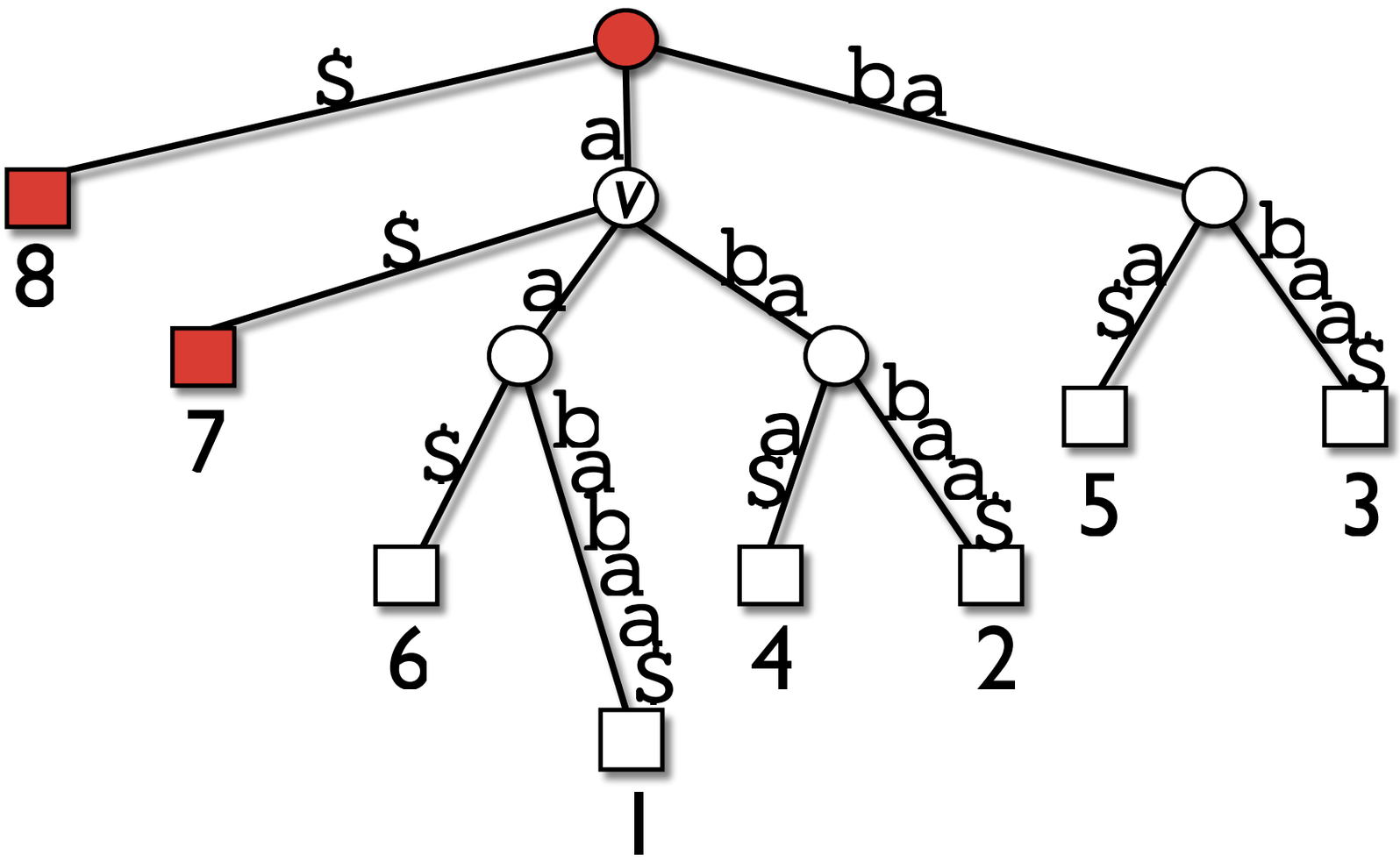
Intervals $[v_l, v_r]$ in H

1. $H[i] \geq \text{SDEPTH}(v)$ $\forall\ v_l < i \leq v_r$
2. $H[v_l] < \text{SDEPTH}(v)$



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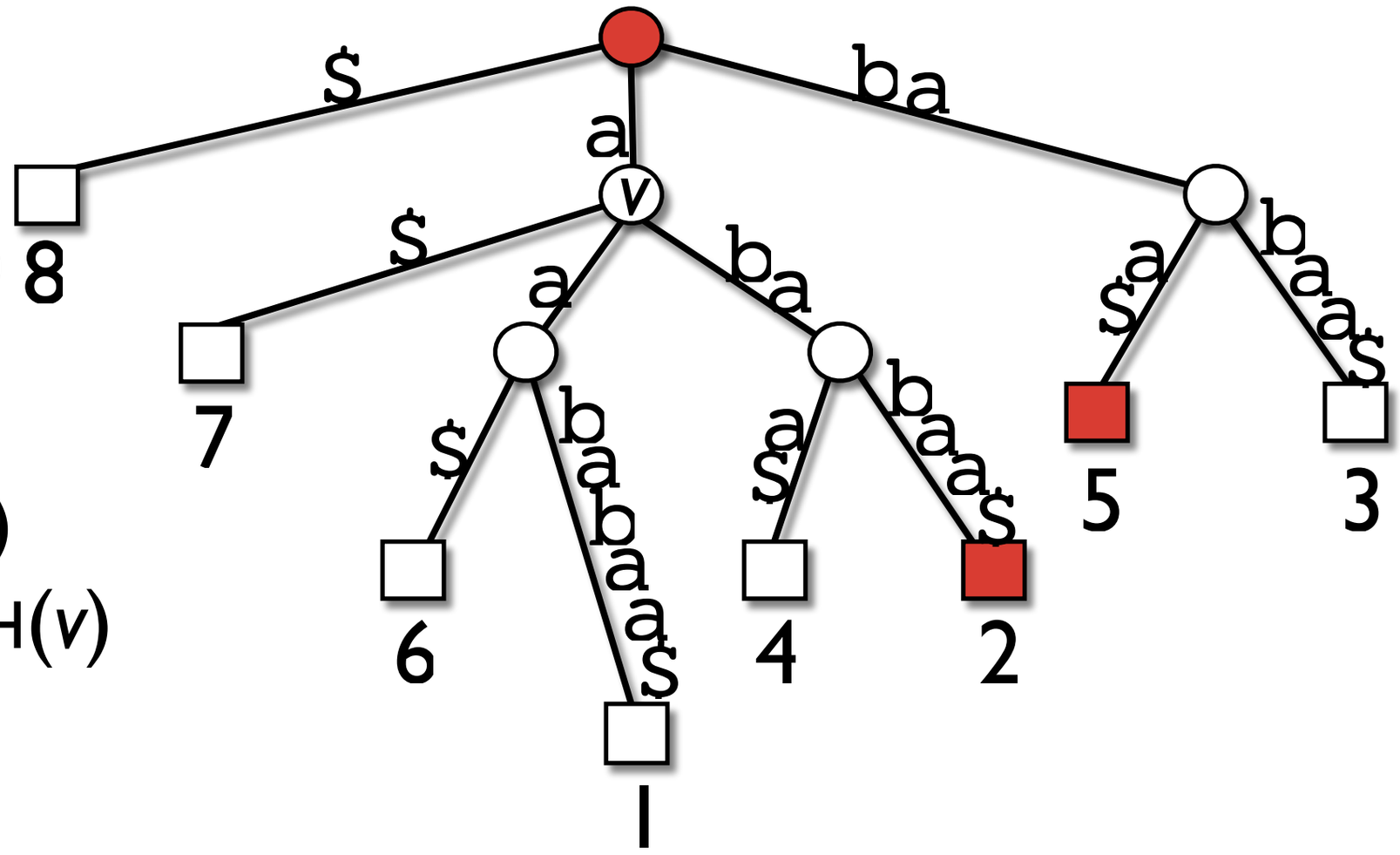
   $H[v_r+1] < \text{SDEPTH}(v)$



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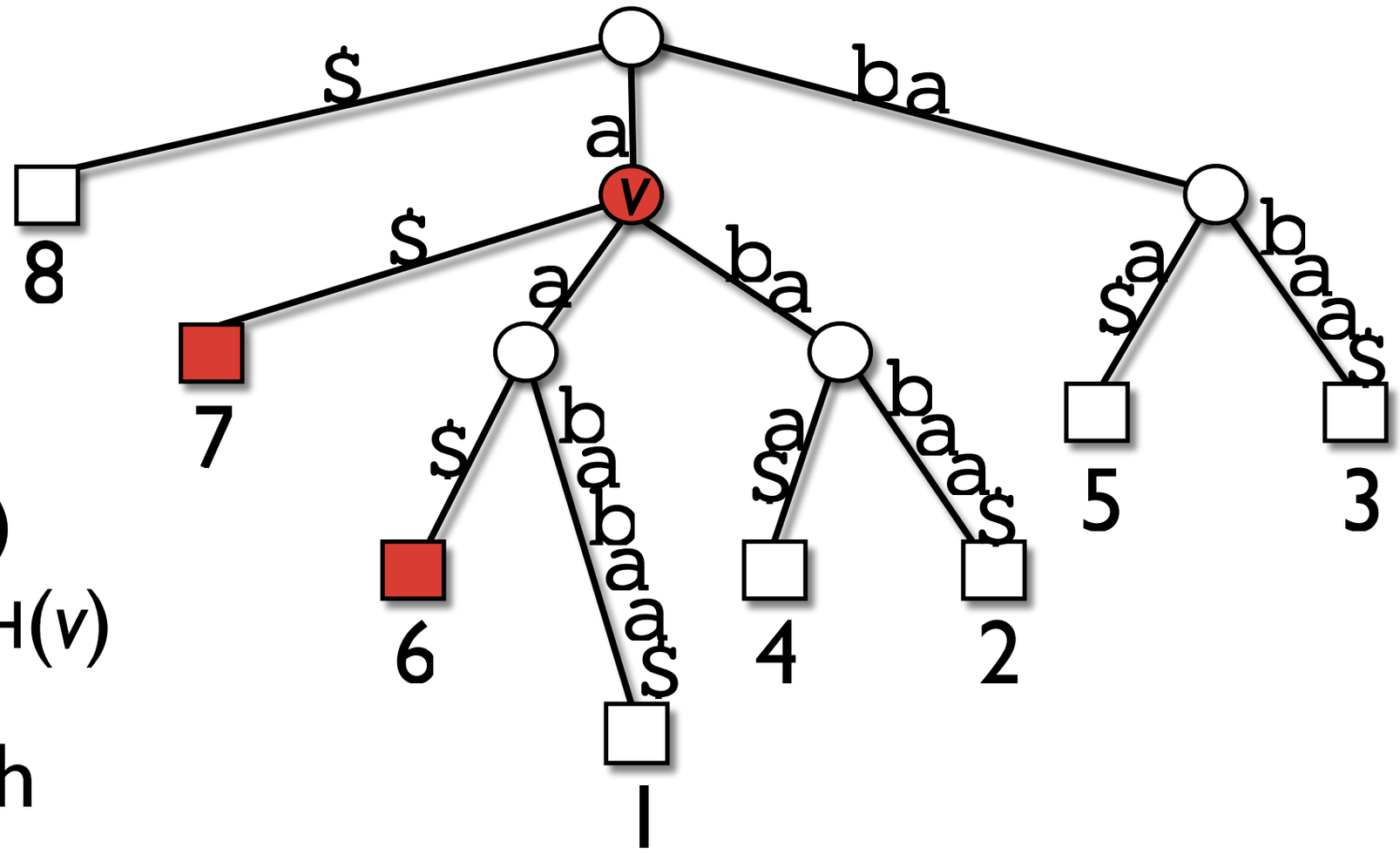
(v_l = 7, v_r = 2)

$H = -1,0,1,2,1,3,0,2$

Intervals $[v_l, v_r]$ in H

1. $H[i] \geq \text{SDEPTH}(v)$ $\forall\ v_l < i \leq v_r$
2. $H[v_l] < \text{SDEPTH}(v)$
   $H[v_r+1] < \text{SDEPTH}(v)$
3. $\exists\ v_l < i \leq v_r$ with
   $H[i] = \text{SDEPTH}(v)$



$A = 8,7,6,1,4,2,5,3$
v_l v_r
$H = -1,0,1,2,1,3,0,2$

Consequences

1. $H[i] \geq \text{SDEPTH}(v)$
   $\forall\ v_l < i \leq v_r$

2. $H[v_l] < \text{SDEPTH}(v)$
   $H[v_r+1] < \text{SDEPTH}(v)$

3. $\exists\ v_l < i \leq v_r$ with
   $H[i] = \text{SDEPTH}(v)$

→ (1) given v_l & v_r: compute i by $i \leftarrow \text{RMQ}_H(v_l+1, v_r)$

$A = 8,7,6,1,4,2,5,3$
(v_l, v_r)

$H = -1,0,1,2,1,3,0,2$
(i; RMQ)

Consequences

1. $H[i] \geq \text{SDEPTH}(v)$
   $\forall\ v_l < i \leq v_r$

2. $H[v_l] < \text{SDEPTH}(v)$
   $H[v_r+1] < \text{SDEPTH}(v)$

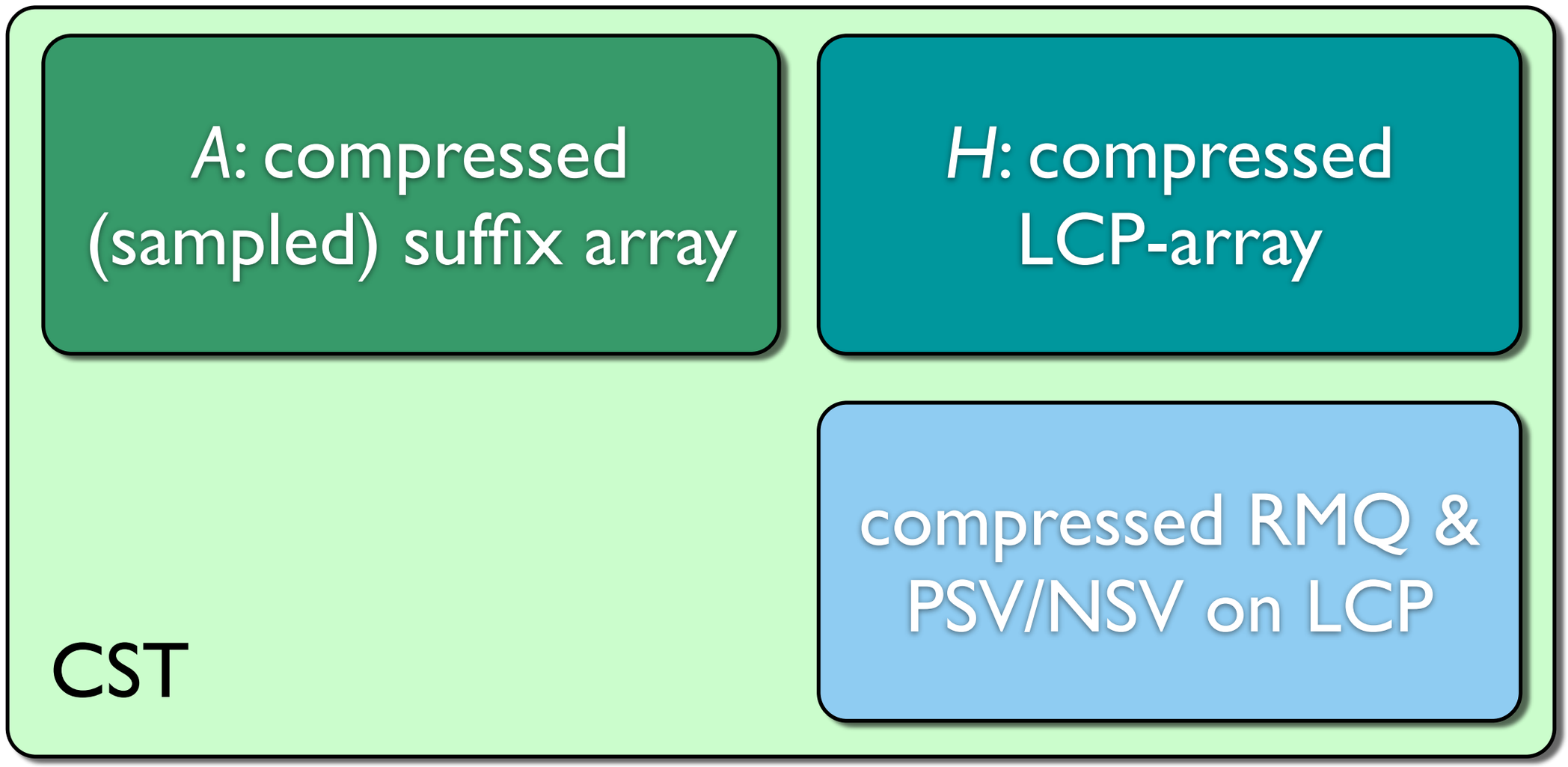
3. $\exists\ v_l < i \leq v_r$ with
   $H[i] = \text{SDEPTH}(v)$

(1) given v_l & v_r: compute i
by $i \leftarrow \text{RMQ}_H(v_l+1, v_r)$

(2) given i: compute
$v_l \leftarrow \text{PSV}_H(i)$
$v_r \leftarrow \text{NSV}_H(i)-1$

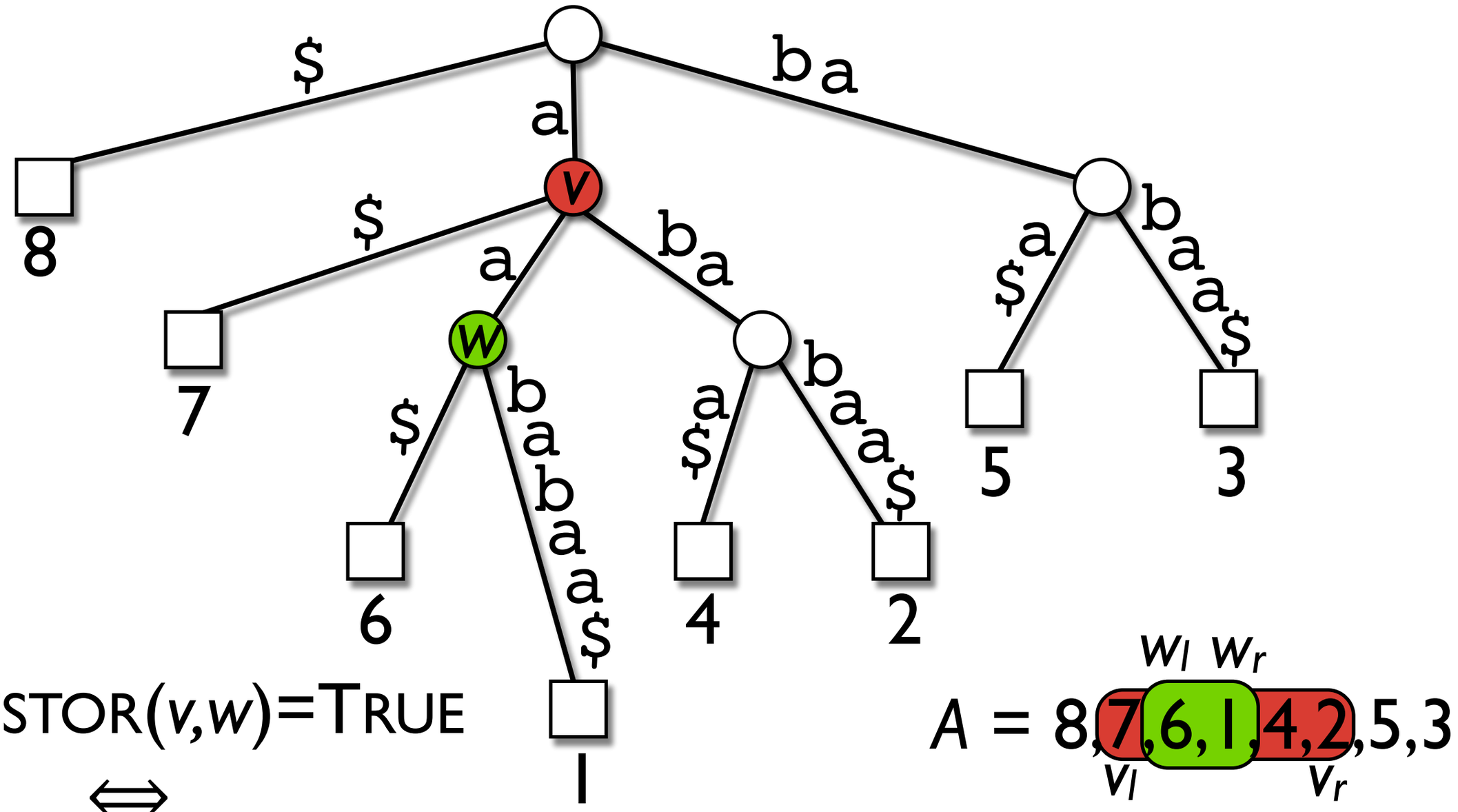
$A = 8,7,6,1,4,2,5,3$
v_l v_r
$H = -1,0,1,2,1,3,0,2$
i
PSV NSV

3 Components of CST



node v represented by interval $[v_l,v_r]$ in H (or A)

IsAncestor(*v*,*w*)



IsAncestor(*v*,*w*)=True

$\Leftrightarrow$

$v_l \leq w_r \leq v_r$

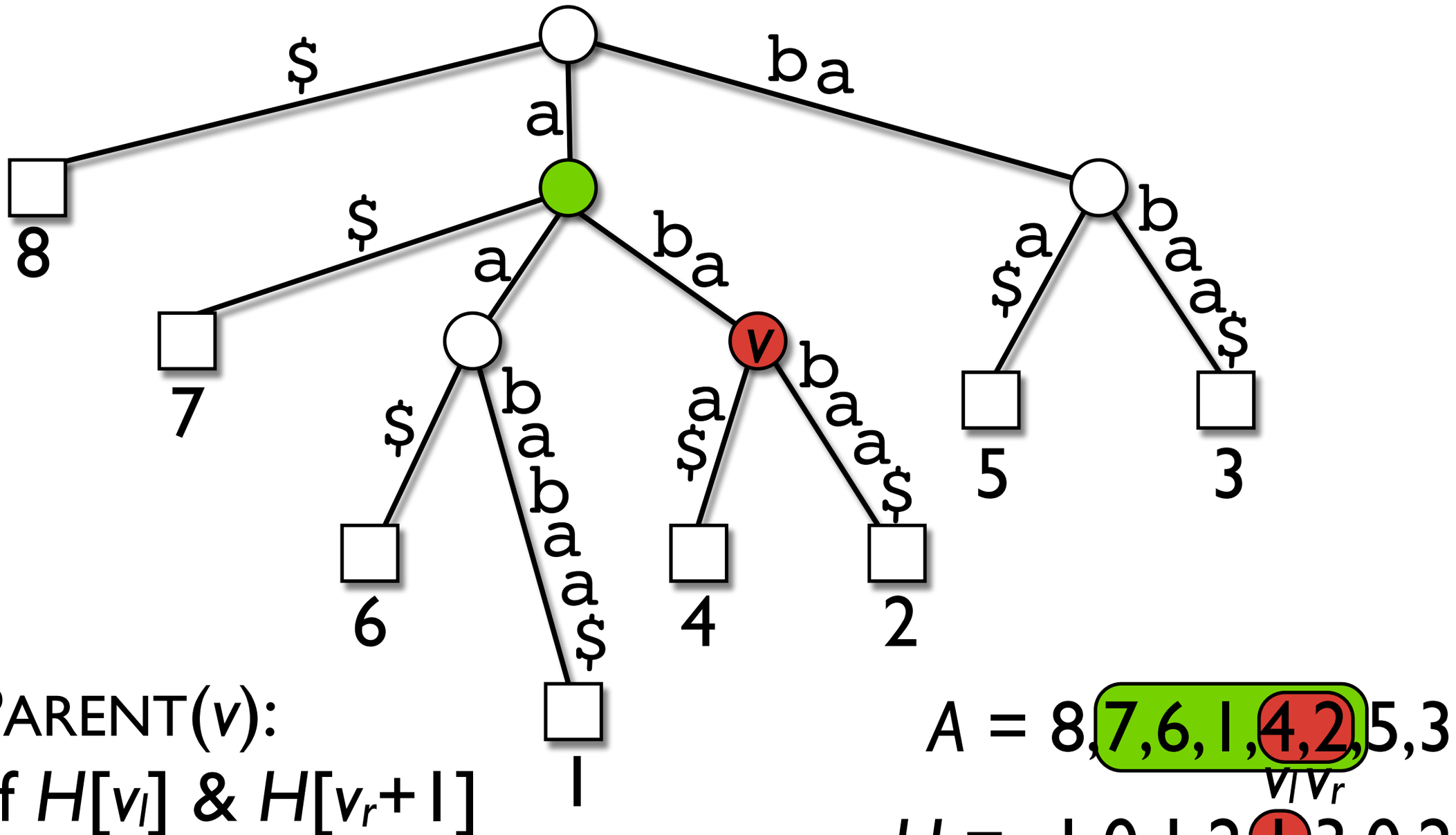
w_l w_r

A = 8,7,6,1,4,2,5,3

v_l v_r

H = -1,0,1,2,1,3,0,2

PARENT(v)



PARENT(v):
larger of $H[v_l]$ & $H[v_r+1]$
is SDEPTH of parent!

A = 8,7,6,1,4,2,5,3
v_l v_r

H = -1,0,1,2,1,3,0,2
PSV NSV

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EDGELABEL(*v*,*i*)	*i*'th letter on the edge leading to *v*
LCA(*v*,*w*)	lowest common ancestor of *v* and *w*

Summary

- Represent ST nodes by **intervals**
- Simulate operations by **RMQ**s and **PSV**s/ **NSV**s on LCP-array

$\Longrightarrow$ suffix and LCP-array **replace** suffix tree

- for a completely compressed ST:
  - compress LCP-array